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Investing in equity mutual funds☆

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Abstract

We construct optimal portfolios of equity funds by combining historical returns on funds and passive indexes with prior views about asset pricing and skill. By including both benchmark and nonbenchmark indexes, we distinguish pricing-model inaccuracy from managerial skill. Modest confidence in a pricing model helps construct portfolios with high Sharpe ratios. Investing in active mutual funds can be optimal even for investors who believe managers cannot outperform passive indexes. Optimal portfolios exclude hot-hand funds even for investors who believe momentum is priced. Our large universe of funds offers no close substitutes for the Fama-French and momentum benchmarks. © 2002 Elsevier Science B.V. All rights reserved.

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1. Introduction

Selecting a portfolio of mutual funds involves a combination of data and judgment. Relying solely on historical fund returns risks overinvesting in funds that have been lucky, especially when funds have short track records. An investor seeking the highest Sharpe ratio can take another extreme and simply combine a riskless investment with a market index fund, accepting completely the CAPM's investment implication. That approach might also be viewed as unappealing because it ignores any information in the data about the inevitable shortcomings of any pricing model, be it the CAPM or another, and it avoids any consideration of evidence that some managers might possess stock-picking skill. How can an investor combine information in the returns data about pricing-model error and managerial skill with his prior judgment about how important those considerations could be?

We develop and implement a framework in which prior views and empirical evidence about pricing models and managerial skill can be formally incorporated into the investment decision. Our framework relies on a set of passive indexes or "assets", consisting of nonbenchmark assets as well as the benchmark assets prescribed by a pricing model. A common interpretation of alpha, the intercept in a regression of the fund's excess return on the benchmarks, is that it represents the skill of the fund's manager in selecting mispriced securities. That interpretation is subject to a number of pitfalls, including a concern that the benchmarks used to define alpha might not price all passive investments. We allow an investor to have prior beliefs about a skill measure that is instead defined as the intercept in a regression of the fund's return on our entire set of passive assets. At the same time, we allow the investor to have prior beliefs about the potential mispricing of the nonbenchmark assets with respect to the benchmarks. In other words, an investor can have prior beliefs that distinguish managerial skill from pricing-model inaccuracy.

Evaluating mutual fund performance is a topic of long-standing interest in the academic literature, but few if any studies have addressed the selection of an optimal portfolio of funds. Instead of using the historical data to estimate performance measures or produce fund rankings, this study uses the data to explore the mutual-fund investment decision. Specifically, from an investment universe of over 500 no-load equity funds, we construct portfolios having the ex ante maximum Sharpe ratio based on a Bayesian predictive distribution that combines the information in historical returns with an investor's prior beliefs, accounting for parameter uncertainty. We entertain priors representing a range of beliefs about managerial skill as well as the accuracy of each of three pricing models: the CAPM, the three-factor Fama-French model, and the four-factor model of Carhart (1997). The last model supplements the three Fama-French benchmarks with a "momentum" factor, the current month's difference in returns between the previous year's best- and worst-performing stocks. Unlike the actual returns on mutual funds, a pricing model's benchmark returns are generally computed without deducting any of the costs associated with implementing the underlying investment strategies. Therefore, while the zero-cost (hypothetical) returns on the passive benchmark and nonbenchmark indexes are used in the modeling and estimation, only mutual funds are assumed eligible for investment.

We find that when the hypothetical benchmarks are recognized as being unavailable for investment, there need not exist close substitutes for them in the universe of mutual funds. For an investor who believes completely in the accuracy of the Fama-French model and precludes managerial skill, the perceived maximum Sharpe ratio is only 66% of what could be achieved by direct investment in that model's benchmarks. For a believer in the Carhart four-factor model, the corresponding value is 54%. Moreover, actively managed funds can be better substitutes for the benchmarks than existing passive funds, so active funds can be selected even by investors who admit no possibility of managerial skill. On the other hand, we find that a "hot-hand" portfolio of the previous year's best-performing mutual funds does not enter the optimal portfolio under any set of prior beliefs about skill or mispricing we consider, even if the investor has complete confidence in the four-factor model that includes a momentum factor as a benchmark.

We also demonstrate that optimal portfolios of mutual funds are influenced substantially by prior beliefs about both managerial skill and pricing models. For example, consider two investors who both rule out managerial skill but believe strongly in different models: one believes in the CAPM while the other embraces a four-factor model. If either investor is forced to hold the portfolio of funds chosen by the other, the resulting ex ante loss is about 60 basis points per month in certainty equivalent return.[1] A possibly flawed pricing model is still useful in identifying optimal portfolios because it allows the model's benchmark assets to supply information about the funds' expected returns. Consider, for example, an investor who rules out skill and whose prior 95% confidence interval for the difference between a fund's expected return and the CAPM-implied value is plus or minus 4% per annum. If that investor is forced to hold the portfolio of funds chosen by an investor who makes no use of a pricing model whatsoever, the certainty equivalent loss is 26 basis points per month. Even for a "completely skeptical" investor who rules out the usefulness of pricing models as well as skill by fund managers, the longer histories of returns on the passive assets provide information about funds' return moments that is valuable in the investment decision. The importance of prior beliefs is demonstrated in ex post out-of-sample results as well. Over the past 20 years, two investors with different prior beliefs about either pricing models or potential managerial skill would have experienced substantially different returns on their portfolios of mutual funds selected each year from the available universe.

This study, given its Bayesian approach, is related to the recent article by Baks et al. (2001), who estimate funds' alphas using informative prior beliefs about alpha. They investigate the degree to which informative priors can preclude an investor from inferring that at least one actively managed fund has a positive alpha. This inference relates to an investment problem of a mutual fund investor who can also earn the hypothetical costless returns on the benchmark indexes. In that setting, if a

[1] The level of risk aversion is set to that of an investor who allocates 100% to a market index if the investment universe contains only that index plus a riskless asset.

given fund's alpha is greater than zero, then combining that fund with a position in the benchmarks produces a higher Sharpe ratio than an investment in the benchmarks alone. Our study differs from the important contribution of Baks et al. (2001) in a number of ways, including that we (i) construct portfolios of funds from the available universe, (ii) do not treat the benchmark returns as directly available for investment, and (iii) distinguish beliefs about the ability of the benchmarks to price passive assets from beliefs about the potential skill of fund managers.

The remainder of the study proceeds as follows. Section 2 discusses the econometric framework, Section 3 presents the results of the investment problem, and Section 4 briefly reviews our conclusions.

2. Framework and methodology

Prior beliefs about pricing models can be useful to someone investing in mutual funds. A pricing model implies that a combination of the model's benchmark assets provides the highest Sharpe ratio within a passive universe. That implication is useful to an investor seeking a high Sharpe ratio, even if the investor has less than complete confidence in the model's pricing accuracy and cannot invest directly in the benchmarks. Prior beliefs about managerial skill are also important in the investment decision. One investor might believe completely in a model's accuracy in pricing passive assets but believe active managers could well possess stock-picking skill. Another investor might be skeptical about the ability of fund managers to pick stocks as well as the ability of academics to build accurate pricing models.

This section develops an econometric framework that allows an investor to combine information in the data with prior beliefs about both pricing and skill. Nonbenchmark assets allow us to distinguish between pricing and skill, and they supply additional information about funds' expected returns. In addition, nonbenchmark assets help account for common variation in funds' returns, making the investment problem feasible using a large universe of funds. The Bayesian econometric framework here is very similar to that in Pástor and Stambaugh (2002), who address performance estimation rather than investment decision making. Consequently, they specify noninformative prior beliefs about the degree of skill a fund manager might possess.

2.1. Mispricing versus skill

Let $r_{N,t}$ denote the $m \times 1$ vector of returns in month t on m nonbenchmark passive assets, and let $r_{B,t}$ denote the vector of returns on the k benchmark assets relevant to a given pricing model. We use "returns" to denote rates of return in excess of a riskless interest rate or payoffs on zero-investment spread positions. Define the multivariate regression

$$r_{N,t} = \alpha_N + B_N r_{B,t} + \varepsilon_{N,t}, \tag{1}$$

where the variance–covariance matrix of $\varepsilon_{N,t}$ is denoted by Σ. Also define the regression of a given fund's return on all p $(= m + k)$ passive assets,

$$r_{A,t} = \delta_A + c'_{AN} r_{N,t} + c'_{AB} r_{B,t} + u_{A,t}, \tag{2}$$

where the variance of $u_{A,t}$ is denoted by σ_u^2. All regression disturbances are assumed to be normally distributed, independently and identically across t, and uncorrelated across funds. In other words, we assume that the nonbenchmark assets account for covariance in fund returns that is not captured fully by the benchmarks.

In both commercial and academic settings, much interest attaches to a fund's alpha, defined as the intercept α_A in the regression

$$r_{A,t} = \alpha_A + \beta_A r_{B,t} + \varepsilon_{A,t}. \tag{3}$$

Alpha is often interpreted as skill displayed by the fund's manager in selecting mispriced securities, but a nonzero alpha need not reflect skill if some passive assets can also have nonzero alphas. In that scenario, a manager could achieve a positive alpha in the absence of any skill simply by starting a new fund that invests in nonbenchmark passive assets with historically positive alphas. To address such concerns, one can expand the set of benchmarks to include more passive assets, even to the point of including all assets available to the manager. Indeed, as observed by Grinblatt and Titman (1989, p. 412), "…the unconditional mean–variance efficient portfolio of assets that are considered tradable by the evaluated investor provides correct inferences about the investor's performance … links between performance measures and particular equilibrium models are not necessary". Chen and Knez (1996) adopt a similar approach in a conditional setting, in that they evaluate funds with respect to a set of passive benchmarks selected without regard to a pricing model: "…we argue that for application purposes, one does not need to rely on asset pricing models to define an admissible performance measure" (p. 515).

In practice, the number of passive assets must be limited in some fashion. Our empirical design includes p passive assets, consisting of k benchmarks and m nonbenchmark assets, and the benchmarks are associated with popular asset pricing models. Suppose one admits the possibility that the benchmarks do not price the nonbenchmark assets exactly, that is $\alpha_N \neq 0$. Then δ_A, the intercept in Eq. (2), is a better measure of skill, in that it is defined with respect to the more inclusive set of passive assets. Of course, that measure might still be nonzero for passive assets omitted from the set of p. The point is simply that inadequacy of δ_A as a skill measure implies inadequacy of α_A, whereas δ_A can be adequate when α_A is not.

The skill measure δ_A is defined with respect to the overall set of p assets, but the investor nevertheless finds it useful to partition that set into k benchmark and m nonbenchmark assets. Even though the investor is unwilling to assume that the k benchmarks price the nonbenchmark assets exactly, he might nevertheless believe that the benchmarks possess some pricing ability. That pricing ability, albeit imperfect, helps the investor identify portfolios with high Sharpe ratios, as illustrated in Section 3.

The prior distributions for the parameters of the regressions in Eqs. (1) and (2) are the same as in Pástor and Stambaugh (2002), with the exception of the prior about managerial skill. The prior for B_N is diffuse, the prior for Σ is inverted Wishart, the prior for σ_u^2 is inverted gamma, and the prior for $c_A = (c'_{AN} c'_{AB})'$, conditional on σ_u^2, is normal. The values for the parameters of the prior distributions are specified using an empirical Bayes procedure described in Pástor and Stambaugh (2002).

Prior beliefs about pricing are specified as follows. Conditional on Σ, the prior for α_N is normal,

$$\alpha_N|\Sigma \sim \mathrm{N}\left(0, \sigma_{\alpha_N}^2\left(\frac{1}{s^2}\Sigma\right)\right), \tag{4}$$

where $\mathrm{E}(\Sigma) = s^2 I_m$. Pástor and Stambaugh (1999) introduce the same type of prior for a single element of α_N, and Pástor (2000) and Pástor and Stambaugh (2000) apply the multivariate version in Eq. (4) to portfolio-choice problems. The investor's beliefs about pricing are characterized by σ_{α_N}, the marginal prior standard deviation of each element in α_N. Specifying $\sigma_{\alpha_N} = 0$ is equivalent to setting $\alpha_N = 0$, corresponding to perfect confidence in the benchmarks' pricing ability. A diffuse prior for α_N corresponds to $\sigma_{\alpha_N} = \infty$. With a nonzero finite value of σ_{α_N}, prior beliefs are centered on the pricing restriction, but some degree of mispricing is entertained. We refer to σ_{α_N} as mispricing uncertainty.

Prior beliefs about managerial skill are specified in a similar manner. Conditional on σ_u^2, the prior for δ_A is normal,

$$\delta_A|\sigma_u^2 \sim \mathrm{N}\left(\delta_0, \left(\frac{\sigma_u^2}{\mathrm{E}(\sigma_u^2)}\right)\sigma_\delta^2\right). \tag{5}$$

The conditional prior variance of δ_A is positively related to σ_u^2 for a reason similar to that given for the corresponding assumption in Eq. (4). If the variation in the fund's return is explained well by that of the benchmarks, so that σ_u^2 is low, then it is less likely that the fund's manager can achieve a large value for δ_A.

We assume that an investor selecting a portfolio of mutual funds generally has informative prior beliefs about the fund managers' ability to achieve a nonzero δ_A. Therefore, we set σ_δ^2, the marginal prior variance of δ_A, to finite values and specify δ_0 as a function of the fund's costs. If a fund manager possesses no skill, then δ_A should simply reflect costs, since the returns on the p passive assets used to define δ_A have no costs deducted. To represent a prior belief that precludes skill, we set $\sigma_\delta = 0$ and specify

$$\delta_0 = -\tfrac{1}{12}(\mathit{expense} \; + \; 0.01 \times \mathit{turnover}), \tag{6}$$

where *expense* is the fund's average annual expense ratio and *turnover* is the fund's average annual reported turnover. Multiplying the latter quantity by 0.01 is equivalent to assuming a round-trip cost per transaction of one percent, approximately the 95 basis points estimated by Carhart (1997) for the average fund in his sample.[2]

[2] Carhart obtains that estimate as the average slope coefficient in monthly cross-sectional regressions of fund return on "modified" turnover, which includes transactions arising from contributions and withdrawals. In forecasting future transactions, it seems reasonable to abstract from growth or shrinkage of the fund and instead view a fund with either no sales or no purchases as a low-turnover fund. Thus, for

When one admits some possibility of skill, the link between turnover and prior expected performance becomes less clear. If the manager does possess skill, then high turnover is likely to be accompanied by positive performance. On the other hand, if the manager possesses no skill, then high turnover can only hurt expected performance. If the investor is uncertain about whether the manager has skill, that is if $\sigma_\delta > 0$, then the relation between expected turnover and expected performance is ambiguous. A similar ambiguity arises with expense ratios. We follow an empirical Bayes approach in specifying how prior expected performance depends on *expense* and *turnover* when $\sigma_\delta > 0$.[3] Specifically, we estimate a cross-sectional regression of estimated δ_A on $\frac{1}{12}expense$ and $\frac{1}{12}turnover$, where the estimate of δ_A is the posterior mean obtained with $\sigma_\delta = \infty$. Across a number of alternative methods for including funds (e.g., minimum history length) and estimating the coefficients (OLS or weighted least squares), we find that the coefficient on $\frac{1}{12}expense$ is consistently about -1 and is at least twice its standard error. In contrast, the coefficient on $\frac{1}{12}turnover$ fluctuates within an interval roughly between -0.005 and 0.005 and is generally less than its standard error.[4] Guided by this result, we specify

$$\delta_0 = -\tfrac{1}{12}\,expense \tag{7}$$

as the prior mean of δ_A when $\sigma_\delta > 0$.

Our framework assumes that funds' sensitivities to passive assets are constant over time. One way of relaxing this assumption is to model these coefficients as linear functions of state variables, as for example in Ferson and Schadt (1996) and Shanken (1990). In such a modification, passive-asset returns scaled by the state variables can be viewed as returns on additional passive assets (dynamic passive strategies), and the approach developed here could be extended to such a setting. Another approach to dealing with temporal variation in parameters could employ data on fund holdings. Daniel et al. (1997) and Wermers (2000), for example, use such data in characteristic-based studies of fund performance.

2.2. Data and the investment problem

The mutual fund data come from the 1998 CRSP Survivor Bias Free Mutual Fund Database.[5,6] Our initial sample contains 2,609 domestic equity mutual funds. We exclude multiple share classes for the same fund as well as funds with only a year or less of available returns. The initial sample is used to obtain the values of the prior

(*footnote continued*)
the value of *turnover* in Eq. (6) we use reported turnover, defined as the minimum of the fund's purchases and sales divided by its average total net assets.

[3] An alternative approach, proposed by Baks et al. (2001), is to specify a prior for performance that is truncated below at a point that reflects expenses as well as an estimate of transaction costs.

[4] Wermers (2000) finds that turnover does not exhibit a significant relation to net performance after adjusting for risk and asset characteristics.

[5] CRSP, Center for Research in Security Prices, Graduate School of Business, The University of Chicago 1999, crsp.com. Used with permission. All rights reserved.

[6] We are grateful to Thomas Knox and the authors of Baks et al. (2001) for providing us with a number of corrections to the CRSP Mutual Fund Database.

parameters in the empirical Bayes procedure mentioned previously. To form the investment universe, we reduce the initial sample of 2,609 funds to the 503 funds that (i) charge no load fee, (ii) exist at the end of 1998, (iii) have at least 36 months of return history under the most recent manager, and (iv) have data on expense ratios and turnover rates. We exclude funds that charge load fees simply because it is not clear how to treat the payment of such fees within the single-period setting implicit in maximizing the Sharpe ratio. For each fund we compute the monthly return in excess of that on a one-month Treasury bill.

Our set of benchmark and nonbenchmark passive assets consists of the eight portfolios used in Pástor and Stambaugh (2002). Monthly returns on all passive assets are available over the period of July 1963 through December 1998. We specify up to four benchmark series. The first three consist of the factors constructed by Fama and French (1993), updated through 1998: MKT, the excess return on a broad market index, and SMB and HML, payoffs on long-short spreads constructed by sorting stocks according to market capitalization and book-to-market ratio.[7] The fourth series, denoted as MOM, is the momentum factor constructed by Carhart (1997). When pricing-model beliefs are centered on the CAPM, then SMB, HML, and MOM become three of the nonbenchmark series. Similarly, when beliefs are centered on the Fama-French model, MOM is then one of the nonbenchmark series. Four additional series are used as nonbenchmark returns with all three pricing models. The first of these, denoted as CMS, is the payoff on a characteristic-matched spread in which the long position contains stocks with low HML betas and the short position contains stocks with high HML betas. The remaining three series are industry portfolios, denoted as IP1, IP2, and IP3. Details on the construction of the passive assets, as well as the reasoning behind choosing this particular set, are provided in Pástor and Stambaugh (2002).

Under various prior beliefs about skill and pricing, we construct portfolios with the highest Sharpe ratio, defined as expected excess return divided by the standard deviation of return. The p passive assets used to define δ_A are included in the econometric specification, but since returns on those assets do not include any implementation costs, only the 503 no-load mutual funds are assumed eligible for investment. In addition, short positions in funds are precluded.

Let R denote the sample data, consisting of returns on passive assets and funds through month T, and let r_{T+1} denote the vector of returns on the funds in month $T+1$. In solving the investment problem, Sharpe ratios are computed using moments of the predictive distribution of the funds' returns,

$$p(r_{T+1}|R) = \int_{\theta} p(r_{T+1}|R, \theta)p(\theta|R)\,\mathrm{d}\theta, \tag{8}$$

where $p(\theta|R)$ is the posterior distribution of the parameter vector, θ.[8] The first two moments of this predictive distribution are derived in the appendix. The fund's

[7] We are grateful to Ken French for supplying these data.

[8] Early applications of Bayesian methods to portfolio choice, using diffuse prior beliefs, include Zellner and Chetty (1965), Klein and Bawa (1976), and Brown (1979). Recent examples, using informative priors, include Pástor (2000) and Pástor and Stambaugh (2000).

history is used only back to the month beginning the most recent manager's tenure, whereas the return histories of the p passive assets begin in July 1963.

A meaningful investment universe can include only those funds that exist at the end of the sample period, December 1998, but this selection criterion raises the issue of survival bias. In particular, under prior beliefs that admit the possibility of skill ($\sigma_\delta > 0$), one might be concerned that the posterior mean of a manager's skill measure δ_A is overstated by a failure to condition on the fund's having survived. Baks et al. (2001) make the interesting observation that, if the priors for skill are independent across funds (as we assume), and if a fund's survival depends only on *realized* return histories, then the posterior distribution of the parameters for the surviving funds is unaffected by conditioning on their survival. In essence, the Bayesian posterior for the parameters conditions on the return histories in any event, and those return histories subsume the information in knowing the fund survived, if survival depends only on realized returns. Like Baks, Metrick, and Wachter, we find the latter view of survival to be plausible, and thus we proceed under that assumption. (For additional discussion of this issue, see also Pástor and Stambaugh, 2002.)

3. Investment results

Table 1 reports weights in the optimal portfolio for investors with various beliefs about managerial skill and mispricing of passive assets under the CAPM. (The weights in each column of Panel A add to 100%.) For convenience, we refer throughout to a portfolio having the highest ex ante Sharpe ratio within a given universe as "optimal". Mispricing uncertainty, σ_{α_N}, is assigned values of zero, 1%, and 2% (per annum), while skill uncertainty, σ_δ, is assigned values of zero, 1%, 3%, and infinity. Tables 2 and 3 report corresponding results for two other pricing models, the Fama-French three-factor model (Table 2) and the Carhart four-factor model (Table 3). Table 4 reports optimal weights for $\sigma_{\alpha_N} = \infty$, in which case the investor makes no use of the pricing models.

3.1. How unique are the selected funds?

In each of Tables 1–4, Panel B compares the optimal portfolios in Panel A to the optimal portfolios constructed from universes of funds that exclude those in the original portfolios. In other words, for each σ_α and σ_δ, the same optimization problem is solved a second time, but the funds selected for the original portfolio are excluded from consideration. From the perspective of an investor with a given set of prior beliefs, this comparison reveals the extent to which there exist close substitutes for the funds selected originally. Panel B reports the correlation between the original and alternative portfolio as well as the difference in certainty equivalent returns. The certainty-equivalent difference is computed for an investor who maximizes the mean–variance objective,

$$C_p = E_p - \tfrac{1}{2}A\sigma_p^2, \tag{9}$$

Table 1
Portfolios with the highest Sharpe ratio under priors for CAPM mispricing and skill of fund managers
The investment universe consists of 503 no-load equity mutual funds with at least three years of return history through December 1998. The benchmark index return MKT is the excess return on the value-weighted stock market. The correlations and certainty-equivalent differences in Panels B and C are computed with respect to the same predictive distribution used to obtain the optimal fund portfolio in the same column. The certainty-equivalent differences are computed with relative risk aversion equal to 2.75.

Mispricing uncertainty (σ_{α_N}) in percent per year:	0	0	0	0	1	1	1	1	2	2	2	2
Skill uncertainty (σ_δ) in percent per year:	0	1	3	∞	0	1	3	∞	0	1	3	∞
Panel A. Portfolio weights (×100)												
Ameristock Mutual Fund	—	—	22	—	—	—	22	—	—	—	20	—
BT Institutional:Equity 500 Index Fund	23	41	—	—	16	23	—	—	—	—	—	—
California Investment S&P 500 Index Fund	53	—	—	—	44	—	—	—	8	—	—	—
Cohen & Steers Realty Shares	—	—	—	—	—	—	—	—	3	5	—	—
Century Shares Trust	—	—	—	—	—	—	—	—	11	—	—	—
DFA AEW Real Estate Securities Portfolio	—	—	—	—	—	—	—	—	18	4	—	—
Elfun Trusts	—	—	—	—	—	5	—	—	—	—	—	—
First American Investment:Real Est Sec/Y	—	—	—	—	—	—	—	—	11	5	—	—
First Funds:Growth and Income Portfolio/I	—	6	—	—	—	5	—	—	—	—	—	—
Gabelli Asset Fund	—	—	—	—	—	—	—	—	—	—	5	—
Galaxy Funds II:Utility Index Fund	—	—	—	—	8	—	—	—	32	18	—	—
IDS Utilities Income Fund/Y	—	—	—	—	—	—	—	—	—	2	—	—
Legg Mason Eq Tr:Value Fund/Navigator	—	—	23	71	—	—	17	67	—	—	4	59
MassMutual Instl Funds:Small Cap Value Eqty/S	—	—	—	—	—	—	—	—	5	—	—	—
Oakmark Fund	—	—	1	—	—	—	4	—	—	—	8	—
Robertson Stephens Inv Tr:Information Age/A	—	—	—	15	—	—	—	11	—	—	—	5
T. Rowe Price Dividend Growth Fund	—	—	—	—	—	—	—	—	—	4	—	—
T. Rowe Price Equity Income Fund	—	30	—	—	—	59	—	—	—	57	—	—
UAM Fds Tr:Heitman Real Estate Portfolio/Inst	—	—	—	—	—	—	—	—	—	3	—	—
Vanguard Index Tr:Extended Market Port/Inv	24	—	—	—	32	—	—	—	12	—	—	—
Vanguard PrimeCap Fund	—	23	4	—	—	9	—	—	—	—	—	—
Weitz Series Fund:Hickory Portfolio	—	—	—	—	—	—	3	2	—	—	6	8
Weitz Series Fund:Value Portfolio	—	—	51	14	—	—	54	20	—	1	57	28
Panel B. Comparison to the portfolio that is optimal for the universe that excludes the funds in the above portfolio												
Correlation (×100)	99	98	93	91	99	98	93	91	95	95	91	92
Certainty-equivalent difference (basis pts./mo.)	0	1	12	43	1	2	12	40	4	5	12	32
Panel C. Comparison to the benchmark index MKT												
Correlation (×100)	100	99	95	93	100	98	94	94	89	92	93	94

where E_p and σ_p^2 denote the mean and variance of the excess return on the investor's overall portfolio (including unrestricted riskless borrowing and lending). Risk-aversion, A, is set to 2.75, which is the level at which an investor would allocate 100% to the benchmark index MKT if the investment universe contained just that single risky position in addition to the riskless asset. The correlation and certainty equivalents are computed using the same predictive distribution used in the portfolio optimization.

Table 2
Portfolios with the highest Sharpe ratio under priors for Fama-French-model mispricing and skill of fund managers
The investment universe consists of 503 no-load equity mutual funds with at least three years of return history through December 1998. The benchmark factors are MKT, the excess return on the value-weighted stock market, SMB, the difference between returns on small and large stocks, and HML, the difference between returns on high and low book-to-market stocks. The correlations and certainty-equivalent differences in Panels B and C are computed with respect to the same predictive distribution used to obtain the optimal fund portfolio in the same column. The certainty-equivalent differences are computed with relative risk aversion equal to 2.75.

Mispricing uncertainty (σ_{α_N}) in percent per year:	0	0	0	0	1	1	1	1	2	2	2	2
Skill uncertainty (σ_δ) in percent per year:	0	1	3	∞	0	1	3	∞	0	1	3	∞
Panel A. Portfolio weights ($\times$100)												
Ameristock Mutual Fund	—	—	10	—	—	—	9	—	—	—	6	—
CGM Realty Fund	—	3	—	—	—	4	—	—	—	4	—	—
Cohen & Steers Realty Shares	—	—	—	—	—	—	—	—	6	5	—	—
Columbia Real Estate Equity Fund	—	—	—	—	—	—	—	—	3	3	—	—
DFA AEW Real Estate Securities Portfolio	13	1	—	—	17	4	—	—	21	8	—	—
DFA Invest Grp:US Large Cap Value Port	2	—	—	—	—	—	—	—	—	—	—	—
First American Investment:Real Est Sec/Y	19	13	—	—	20	15	—	—	21	15	—	—
Galaxy Funds II:Utility Index Fund	8	5	—	—	13	11	—	—	22	18	—	—
Legg Mason Eq Tr:Total Return Fund/Navigator	40	10	—	—	34	6	—	—	21	—	—	—
Legg Mason Eq Tr:Value Fund/Navigator	—	—	—	44	—	—	—	43	—	—	—	41
Mutual Discovery Fund/Z	18	39	37	26	15	35	34	24	6	25	28	18
Oakmark Fund	—	—	2	—	—	—	3	—	—	—	4	—
T. Rowe Price Equity Income Fund	—	29	7	—	—	25	8	—	—	17	12	—
UAM Fds Tr:Heitman Real Estate Portfolio/Inst	—	—	—	—	—	—	—	—	—	3	—	—
Weitz Series Fund:Hickory Portfolio	—	—	—	1	—	—	—	3	—	—	—	6
Weitz Series Fund:Value Portfolio	—	—	45	29	—	—	47	31	—	—	50	35
Panel B. Comparison to the portfolio that is optimal for the universe that excludes the funds in the above portfolio												
Correlation ($\times$100)	95	95	92	93	94	95	92	93	93	94	91	93
Certainty-equivalent difference (basis pts./mo.)	9	11	19	25	8	11	18	24	11	10	15	23
Panel C. Comparison to the combination of the benchmark indexes MKT, SMB, and HML having the highest Sharpe ratio												
Correlation ($\times$100)	75	74	66	55	75	75	65	55	73	74	64	54

In some cases there are close substitutes for the original funds. For example, when an investor believes completely in the CAPM and rules out managerial skill, the optimal portfolio is a combination of market-index funds (Table 1, first column). Not surprisingly, our fund universe contains other index funds in addition to those selected, and thus the alternative portfolio is highly correlated with the original and achieves a virtually identical certainty equivalent.

In many other cases the funds originally selected are more unique. For example, an investor with $A = 2.75$ who believes completely in the four-factor model and rules out skill loses 25 basis points per month if forced to hold the alternative portfolio

Table 3

Portfolios with the highest Sharpe ratio under priors for four-factor-model mispricing and skill of fund managers

The investment universe consists of 503 no-load equity mutual funds with at least three years of return history through December 1998. The benchmark factors are MKT, the excess return on the value-weighted stock market, SMB, the difference between returns on small and large stocks, HML, the difference between returns on high and low book-to-market stocks, and MOM, the difference between returns on stocks with high and low returns over the previous year (excluding the most recent month). The correlations and certainty-equivalent differences in Panels B and C are computed with respect to the same predictive distribution used to obtain the optimal fund portfolio in the same column. The certainty-equivalent differences are computed with relative risk aversion equal to 2.75.

Mispricing uncertainty (σ_{α_N}) in percent per year:	0	0	0	0	1	1	1	1	2	2	2	2
Skill uncertainty (σ_δ) in percent per year:	0	1	3	∞	0	1	3	∞	0	1	3	∞
Panel A. Portfolio weights ($\times$100)												
Alpine US Real Estate Equity Fund/Y	—	1	—	—	—	1	—	—	—	1	—	—
CGM Realty Fund	1	10	6	—	—	9	6	—	—	8	5	—
Cohen & Steers Realty Shares	14	14	9	—	14	14	9	—	14	14	10	—
Columbia Real Estate Equity Fund	11	12	6	—	10	12	6	—	9	11	7	—
DFA AEW Real Estate Securities Portfolio	28	19	—	—	27	19	—	—	26	18	—	—
First American Investment:Real Est Sec/Y	20	16	4	—	19	16	5	—	18	15	5	—
Gabelli Asset Fund	—	—	8	—	—	—	6	—	—	—	1	—
Galaxy Funds II:Utility Index Fund	14	11	—	—	17	13	—	—	21	18	—	—
Legg Mason Eq Tr:Value Fund/Navigator	—	—	—	48	—	—	—	46	—	—	—	44
Lindner/Ryback Small Cap Fund/Investor	—	—	—	2	—	—	—	2	—	—	—	1
Morgan Stanley Dean Witter Ist:US Real Est/A	—	—	2	—	—	—	2	—	—	—	3	—
Mutual Discovery Fund/Z	—	—	4	7	—	—	3	6	—	—	2	5
T. Rowe Price Dividend Growth Fund	—	—	14	—	—	—	15	—	—	—	16	—
UAM Fds Tr:Heitman Real Estate Portfolio/Inst	13	16	9	—	12	16	9	—	11	15	9	—
Weitz Series Fund:Hickory Portfolio	—	—	—	10	—	—	—	11	—	—	1	12
Weitz Series Fund:Value Portfolio	—	—	38	34	—	—	39	35	—	—	40	37
Panel B. Comparison to the portfolio that is optimal for the universe that excludes the funds in the above portfolio												
Correlation ($\times$100)	87	89	93	92	87	89	93	92	88	89	93	92
Certainty-equivalent difference (basis pts./mo.)	25	24	19	23	26	24	19	22	24	24	20	24
Panel C. Comparison to the combination of the benchmark indexes MKT, SMB, HML, and MOM having the highest Sharpe ratio												
Correlation ($\times$100)	61	62	50	30	61	62	50	30	61	62	50	30

that excludes the funds originally selected (Table 3, first column). The correlation of that alternative portfolio with the original is only 0.87. As will be discussed later, the original portfolio in that case is heavily invested in real estate funds. An investor who believes completely in the Fama-French model and rules out skill would select a portfolio that contains value-oriented funds as well as some real estate funds (Table 2, first column). If forced to choose an alternative portfolio, that investor loses nine basis points per month in certainty equivalent, and the alternative portfolio has a correlation of 0.95 with the original. For larger values of σ_δ, or when no pricing

Table 4

Portfolios with the highest Sharpe ratio under priors for skill of fund managers and no use of a pricing model ($\sigma_{\alpha_N} = \infty$)

The investment universe consists of 503 no-load equity mutual funds with at least three years of return history through December 1998. The benchmark factors are MKT, the excess return on the value-weighted stock market, SMB, the difference between returns on small and large stocks, HML, the difference between returns on high and low book-to-market stocks, and MOM, the difference between returns on stocks with high and low returns over the previous year (excluding the most recent month). The correlations and certainty-equivalent differences in Panels B–E are computed with respect to the same predictive distribution used to obtain the optimal fund portfolio in the same column. The certainty-equivalent differences are computed with relative risk aversion equal to 2.75.

Skill uncertainty (σ_δ) in percent per year:	0	1	2	3	∞
Panel A. Portfolio weights ($\times$100)					
CGM Realty Fund	—	2	2	1	—
Cohen & Steers Realty Shares	13	14	13	11	—
Cappiello-Rushmore Trust:Utility Income Fund	—	1	—	—	—
Columbia Real Estate Equity Fund	7	9	9	7	—
DFA AEW Real Estate Securities Portfolio	20	14	2	—	—
First American Investment:Real Est Sec/Y	14	12	11	6	—
Galaxy Funds II:Utility Index Fund	37	32	21	8	—
IDS Utilities Income Fund/Y	—	5	5	—	—
Legg Mason Eq Tr:Value Fund/Navigator	—	—	—	—	32
Morgan Stanley Dean Witter Ist:US Real Est/A	—	—	2	5	3
Oakmark Fund	—	—	—	—	2
T. Rowe Price Dividend Growth Fund	—	—	—	6	—
UAM Fds Tr:Heitman Real Estate Portfolio/Inst	9	12	11	8	—
Weitz Partners Value Fund	—	—	—	—	2
Weitz Series Fund:Hickory Portfolio	—	—	—	6	18
Weitz Series Fund:Value Portfolio	—	—	24	42	43
Panel B. Comparison to the portfolio that is optimal for the universe that excludes the funds in the above portfolio					
Correlation ($\times$100)	89	89	90	92	92
Certainty-equivalent difference (basis pts./mo.)	27	27	22	21	24
Panel C. Correlation ($\times$100) *with the portfolio having the highest Sharpe ratio that combines the benchmark factors shown*					
MKT	74	76	82	87	94
MKT, SMB, HML	66	66	66	64	51
MKT, SMB, HML, MOM	59	60	57	50	31
Panel D. Comparison to the portfolio that is optimal under $\sigma_\delta = 0$					
Correlation ($\times$100)	100	100	97	89	71
Certainty-equivalent difference (basis pts./mo.)	0	1	6	23	133
Panel E. Comparison to the portfolio that is optimal when expected returns equal sample means					
Correlation ($\times$100)	68	69	75	80	91
Certainty-equivalent difference (basis pts./mo.)	477	440	367	310	187

model is used, the certainty-equivalent differences from the original portfolios are typically at least 20 basis points per month, and the correlations are about 0.93 or less.

3.2. How important are beliefs about pricing models and skill?

We stated earlier that a pricing model, even if not believed completely, helps an investor identify portfolios with high Sharpe ratios. Consider the CAPM, for example. For an investor who believes completely in the CAPM and rules out skill, the ideal investment's return is perfectly correlated with the benchmark index return MKT. Indeed, as shown in Panel C of Table 1, the portfolio of funds in the first column essentially possesses that feature. A value of $\sigma_{\alpha_N} = 1\%$ means that, before examining the data, the investor assigns about a 5% probability to the prospect that the expected return on a given nonbenchmark passive asset violates its CAPM prediction by more than 200 basis points per annum in either direction. With that degree of mispricing uncertainty but the same belief about skill, the optimal portfolio is still essentially composed of market index funds and has a correlation with MKT that rounds to 1.00. With twice as much mispricing uncertainty ($\sigma_{\alpha_N} = 2\%$), the correlation with MKT is 0.89, which is still considerably higher than the value of 0.74 obtained when no pricing model is used (Table 4, Panel C, first entry).

The CAPM continues to influence portfolio choice when the investor admits the possibility of managerial skill. A value of $\sigma_\delta = 1\%$ means that, before examining a given fund's track record, the investor assigns about a 2.5% probability to the prospect that the fund's manager generates a positive skill measure gross of expenses of at least 200 basis points per year. (Of course, given the symmetry of our prior, the investor assigns the same probability to a negative skill measure of that magnitude, but the left tail is unimportant with short sales precluded.) With that amount of skill uncertainty, the CAPM can still help the investor construct the portfolio with the highest Sharpe ratio, even with some uncertainty about the CAPM's ability to price passive assets. When $\sigma_\delta = 1\%$, the optimal portfolio has a correlation of 0.92 with MKT when $\sigma_{\alpha_N} = 2\%$ (Table 1), as compared to a correlation of only 0.76 when no model is used. With three times as much skill uncertainty ($\sigma_\delta = 3\%$), the optimal portfolio's correlation with MKT is 0.93 when $\sigma_{\alpha_N} = 2\%$ and 0.87 when the model is not used. That is, even with a substantial degree of willingness to accept the possibility of managerial skill and only modest confidence in the CAPM, the investor's portfolio selection is still influenced by the pricing model.

Note that, holding σ_{α_N} constant, the correlation of the optimal fund portfolio with MKT can increase or decrease as a function of σ_δ. When $\sigma_\delta = 0$, funds are chosen primarily for their exposures to passive assets. As σ_δ increases, more weight is put on the funds with high realized returns, and those funds can be more or less correlated with MKT than the funds chosen for $\sigma_\delta = 0$. Panel C of Table 1 provides examples of both possibilities: the correlations go up with σ_δ when $\sigma_{\alpha_N} = 2\%$, but they go down with σ_δ when $\sigma_{\alpha_N} = 0$.

Portfolio choice is influenced by beliefs in the other pricing models in similar ways as noted above for the CAPM. For an investor who believes completely in the Fama-

French model and rules out skill, the ideal investment, if available, would have a return perfectly correlated with the combination of MKT, SMB, and HML having the highest Sharpe ratio. As will be discussed below, a close substitute for such an investment is not available within our fund universe, but a belief in that pricing model nevertheless plays a role in the selection of funds. For example, the correlation between the portfolio in the first column of Table 2 and the ideal combination of the three Fama-French factors is 0.75, whereas the portfolio chosen with the same beliefs about skill but no use of a pricing model has a correlation of only 0.66 with that same combination of the three factors (Table 4, Panel C). As before, the pricing model continues to play a role in portfolio choice as one's belief in it becomes less than dogmatic ($\sigma_\alpha > 0$) and the possibility of skill is entertained ($\sigma_\delta > 0$).

Panel A of Table 5 compares portfolios formed with the same σ_{α_N} and σ_δ but under different pricing models. In comparing portfolios obtained under different specifications, one portfolio is designated as optimal and the other as suboptimal, where the suboptimal portfolio is optimal under the alternative specification. We compare the certainty equivalent for the optimal portfolio, C_o, to the certainty equivalent for a suboptimal portfolio, C_s. Both certainty equivalents are computed using the predictive moments obtained under the prior beliefs associated with the optimal portfolio. The difference between any two models ranges between one and 61 basis points per month, depending on the prior uncertainty about mispricing and skill.[9] In general, sample averages receive more weight in estimating expected returns when one's prior beliefs about pricing and skill become less informative. As mispricing uncertainty increases, the portfolios formed with beliefs centered on different pricing models become more alike: the certainty-equivalent difference drops and the correlation increases. An increase in skill uncertainty also tends to make the cross-model difference less important, although not monotonically. The largest certainty-equivalent differences tend to occur between the CAPM and the four-factor model when σ_{α_N} and σ_δ are small. The smallest differences occur between the Fama-French and four-factor models when σ_δ is large. When σ_{α_N} and σ_δ are both 1% or less, however, the certainty-equivalent difference between those two models is at least 19 basis points per month.

In Panel B of Table 5, the optimal portfolio under a given set of beliefs about skill and mispricing is compared to the portfolio selected by an investor who rules out any ability of academics to build models and any skill of portfolio managers to pick stocks. The portfolio of this "completely skeptical" investor, for whom $\sigma_{\alpha_N} = \infty$ and $\sigma_\delta = 0$, is designated as the suboptimal portfolio in computing the pairwise comparisons described previously. (Its weights are given in the first column of Table 4.) Suppose one forces that portfolio to be held by an investor who has a modest degree of confidence in the CAPM, say $\sigma_{\alpha_N} = 2\%$, and who admits some

[9] The reported certainty-equivalent difference is actually the average of two differences, one for each of the two pricing models designated as producing the optimal portfolio. The correlation reported in Panel A is similarly the average of two values, one for the predictive distribution associated with each model. Averaging in this fashion treats the pricing models symmetrically, although generally the two values being averaged are close to each other.

Table 5

Comparisons of portfolios of no-load funds formed under various prior beliefs about manager skill and pricing models

All portfolios being compared are formed from an investment universe of 503 no-load equity mutual funds with at least three years of return history through December 1998. The pricing models considered are the Capital Asset Pricing Model of Sharpe (1964) and Lintner (1965), the three-factor Fama-French (1993) model, and the four-factor model of Carhart (1997), which adds a momentum factor to the Fama-French model. All of the reported correlations and certainty-equivalent differences are computed using the predictive distribution formed under the prior mispricing uncertainty (σ_{α_N}) and skill uncertainty (σ_δ) in the column heading. The certainty-equivalent difference is computed with relative risk aversion equal to 2.75.

Mispricing uncertainty (σ_{α_N}) in percent per year:	0	0	0	0	1	1	1	1	2	2	2	2
Skill uncertainty (σ_δ) in percent per year:	0	1	3	∞	0	1	3	∞	0	1	3	∞
Panel A. Comparison of the portfolios formed with the same σ_{α_N} and σ_δ under different pricing models												
Certainty-equivalent difference (basis points per month)												
CAPM versus Fama-French	26	25	19	28	23	19	14	21	9	8	8	10
CAPM versus four-factor	59	61	34	18	50	51	28	13	24	27	19	5
Fama-French versus four-factor	24	29	19	3	19	23	17	2	10	13	13	1
Correlation ($\times$100)												
CAPM versus Fama-French	87	89	93	94	87	91	95	96	97	96	97	98
CAPM versus four-factor	73	71	91	96	76	75	92	97	94	89	93	99
Fama-French versus four-factor	89	84	94	99	92	88	94	99	97	94	94	100
Panel B. Comparison of the optimal portfolio to the portfolio of a "completely skeptical" investor ($\sigma_{\alpha_N} = \infty$ and $\sigma_\delta = 0$)												
Certainty-equivalent difference (basis points per month)												
CAPM	71	74	138	299	53	57	121	275	26	29	89	232
Fama-French	28	35	94	227	22	27	85	217	11	15	67	196
Four-factor	4	6	35	153	3	5	34	151	2	3	31	147
Correlation ($\times$100)												
CAPM	73	71	70	60	77	74	71	62	95	90	73	65
Fama-French	87	82	73	69	91	86	73	69	96	93	73	70
Four-factor	98	97	87	69	98	97	87	69	99	98	88	69
Panel C. Comparison of the optimal portfolio to the portfolio that is optimal when expected returns equal sample means												
Certainty-equivalent difference (basis points per month)												
CAPM	393	356	259	172	395	358	262	171	404	367	270	171
Fama-French	414	380	281	171	417	383	282	172	426	390	285	174
Four-factor	465	428	302	185	466	429	303	185	468	431	305	185
Correlation ($\times$100)												
CAPM	95	95	94	94	94	94	93	95	83	87	89	95
Fama-French	81	82	84	93	80	82	84	93	77	79	84	93
Four-factor	67	67	82	94	67	68	82	94	68	68	81	93

possibility of managerial skill, say $\sigma_\delta = 1\%$. Then that investor suffers a certainty-equivalent loss of 29 basis points per month, or about 3.5% per year. With beliefs centered around the Fama-French model but again with $\sigma_{\alpha_N} = 2\%$ and $\sigma_\delta = 1\%$, the certainty-equivalent loss falls to 15 basis points per month. When skill uncertainty is 1% or less, complete belief in the four-factor model produces a portfolio quite close to that obtained with no use of the model at all, with a certainty-equivalent difference of six basis points or less and a correlation of at least 0.97. As an investor's willingness to accept the prospect of managerial skill increases, so does the certainty-equivalent loss if forced to hold the portfolio of the completely skeptical investor. With $\sigma_\delta = 3\%$, for example, the loss is between 31 and 89 basis points per month with modest confidence ($\sigma_{\alpha_N} = 2\%$) in one of the three pricing models. With no use of a pricing model, the loss is 23 basis points, as reported in Panel D of Table 4.

Even with no belief in a pricing model and no preconceived limit on the magnitude of likely managerial skill, that is when both σ_{α_N} and σ_δ are infinitely large, the investor is generally ill-advised in using a fund's historical average return as the input for its expected return. A number of papers, including Jobson and Korkie (1980), show that using sample averages in mean–variance portfolio optimization typically results in portfolios with poor out-of-sample behavior. If the fund's history is shorter than those of the passive assets, then the histories of the passive assets provide additional information about the fund's expected return (essentially as in Stambaugh, 1997). Under the above prior beliefs, the certainty-equivalent loss of holding the portfolio constructed using sample averages instead of holding the portfolio constructed using that additional information about expected returns is 187 basis points per month, or more than 22% annually (Panel E of Table 4). The predictive covariance matrix obtained when $\sigma_{\alpha_N} = \infty$ and $\sigma_\delta = \infty$ is used to construct both portfolios. As prior beliefs about pricing or skill become informative, the loss incurred by holding the portfolio based on sample averages becomes even greater, as is apparent in Panel C of Table 5. The magnitude of the loss is amplified by a leverage effect. Unlike the portfolio of our Bayesian investor, the portfolio of an investor who uses sample means is highly levered, since the latter investor borrows heavily to invest in the funds with high realized returns. Note that while the differences in leverage affect the certainty equivalent loss, they have no effect on the correlation between the two portfolios.

Recall that prior beliefs about skill are centered at a value reflecting a fund's costs. The results appear to be fairly insensitive to this specification, in that setting δ_0 to zero instead of the negative values in Eq. (6) or (7) produces optimal portfolios (unreported) that are generally very similar to those in Tables 1–4. A notable exception occurs with dogmatic beliefs in the CAPM and no skill ($\sigma_\alpha = \sigma_\delta = 0$), where the investor essentially constructs the portfolio of funds that best mimics the market portfolio. In that case, the optimal portfolio with δ_0 as specified in Eq. (6) includes only three market index funds (Table 1, first column), whereas the optimal portfolio with $\delta_0 = 0$ contains over 40 funds. This difference is easily explained. Suppose the portfolio already includes a fund that is a good proxy for the market. With $\delta_0 = 0$, there is no penalty for adding more funds that reduce tracking error a bit further, whatever those funds' expenses. With $\delta_0 < 0$, the small reduction in tracking error from adding funds is more than offset by the penalty arising from those funds' somewhat higher expenses. Therefore, the optimal portfolio in the latter case has fewer funds and a slightly larger tracking error, but the optimal portfolio in either case has a correlation with the market that rounds to 100%.

3.3. Who should buy actively managed funds?

One might presume that actively managed funds should be purchased only by those investors who admit some possibility that active fund managers possess stock-picking skill. For investors presented with our universe of 503 no-load funds, that need not be the answer. An investor who believes completely in the CAPM and admits no possibility of managerial skill does indeed invest only in market-index funds (Table 1). As the investor's beliefs depart from complete confidence in the CAPM, however, actively managed funds enter the optimal portfolio even if the investor still adheres to a belief that managerial skill is impossible. If one can invest directly and costlessly in the p passive assets used to define the skill measure δ_A, then indeed long positions in funds arise only when positive δ_A's are thought possible. With positive δ_A's precluded, one simply combines the passive assets to obtain the highest Sharpe ratio. Baks et al. (2001) essentially pose their active management question in that context. If instead the p passive assets are not available for investment, as in our setup, perfect substitutes for them need not exist in the mutual fund universe, let alone in its passively managed subset. As a result, some actively managed funds can become attractive even to investors who admit no chance of managerial skill.

A striking example of the above possibility occurs in the first column of Table 2. The investor in that case believes completely in the Fama-French model and in no chance of managerial skill. Nevertheless, the bulk of that investor's optimal portfolio is allocated to actively managed value funds and real estate specialty funds: Legg Mason Total Return, Mutual Discovery, First American Investment Real Estate Securities and DFA AEW Real Estate Securities. Table 6 reports posterior means of the intercept and selected slopes in Eq. (2) for all funds that receive at least a 10% allocation in any of the porfolios in Tables 1–4. The selection of the above funds has nothing to do with their having superior historical performance. In fact, three of the four funds listed above have negative $\hat{\delta}_A$'s. With $\sigma_{\alpha_N} = \sigma_\delta = 0$, the expected returns on these funds, gross of costs, are assumed to conform exactly to the Fama-French model. The presence of these funds in the optimal portfolio is instead driven by their risk characteristics. Note, for example, that all four funds have significantly positive slopes on HML.

3.4. Hot hands?

To the universe of 503 no-load funds, we also add a portfolio of funds with high recent returns, motivated by previous research indicating short-run persistence in fund performance.[10] At the end of each year, starting with December 1962, we sort all no-load equity funds by their total returns over the previous 12 months (including only funds with returns reported for those months) and form the equally weighted "hot-hand" portfolio of the top 10%. As Carhart (1997) observes, such a portfolio

[10] See, for example, Grinblatt and Titman (1992), Hendricks et al. (1993), and Brown and Goetzmann (1995).

Table 6
Coefficients in regressions of fund returns on the passive-asset returns
The table reports posterior means (multiplied by 100) of the intercept (δ_A) and selected slope coefficients in a regression of the fund's return on the returns of eight passive assets. The passive assets are CMS, a spread between stocks with high and low HML betas but with both legs matched in terms of market capitalization (size) and book-to-market ratios, IP1–IP3, three portfolios formed by applying principal-component analysis to a set of 20 industry portfolios, MOM, the difference between returns on stocks with high and low returns over the previous year (excluding the most recent month), SMB, the difference between returns on small and large stocks, HML, the difference between returns on high and low book-to-market stocks, and MKT, the excess return on the value-weighted stock market. The "t-statistics" used to determine significance are calculated by dividing the coefficient's posterior mean by its posterior standard deviation.

	δ	MOM	SMB	HML	MKT
Ameristock Mutual Fund	0.39*	−6	−34**	8	90**
BT Institutional:Equity 500 Index Fund	0.09	−2	−25**	−2	96**
CGM Realty Fund	−0.14	19*	58**	63**	109**
California Investment S&P 500 Index Fund	0.07	−2	−24**	−2	96**
Century Shares Trust	−0.45	9	8	43**	132**
Cohen & Steers Realty Shares	−0.22	28**	69**	53**	102**
Columbia Real Estate Equity Fund	−0.26	22**	46**	61**	94**
DFA AEW Real Estate Securities Portfolio	−0.59*	23**	57**	64**	99**
First American Investment:Real Est Sec/Y	−0.29	16*	47**	71**	106**
Galaxy Funds II:Utility Index Fund	−0.57*	13**	13	63**	140**
Legg Mason Eq Tr:Total Return Fund/Navigator	−0.16	−6	10	67**	91**
Legg Mason Eq Tr:Value Fund/Navigator	0.84**	−5	−24*	−14	82**
Mutual Discovery Fund/Z	0.34	−10	39**	57**	64**
Robertson Stephens Inv Tr:Information Age/A	1.13	8	32	−133**	75*
T. Rowe Price Dividend Growth Fund	0.21*	4	−3	20**	78**
T. Rowe Price Equity Income Fund	0.16*	−6**	0	36**	70**
UAM Fds Tr:Heitman Real Estate Portfolio/Inst	−0.32	31**	70**	52**	107**
Vanguard Index Tr:Extended Market Port/Inv	−0.08	6**	54**	−3	103**
Vanguard PrimeCap Fund	0.44**	−3	9	−32**	68**
Weitz Series Fund:Hickory Portfolio	0.56	0	61**	18	108**
Weitz Series Fund:Value Portfolio	0.41*	−4	19**	18*	72**
Hot-Hand Portfolio	−0.00	15**	48**	−8*	93**

*The coefficients are statistically significant at the 5% level.
**The coefficients are statistically significant at the 1% level.

has a positive sensitivity to the momentum factor MOM, which is confirmed by the results in Table 6. The hot-hand portfolio appears in the last row, and the posterior mean of its coefficient on MOM is 0.15 (with a "t-statistic" of 8.7, calculated as the posterior mean divided by posterior standard deviation). This portfolio does not enter any of the optimal portfolios reported in Tables 1–4. When the hot-hand portfolio is formed from the universe of all funds, as opposed to just the no-load subset, the posterior mean of its coefficient on MOM is 0.21 with a t-statistic of 12.4. That version of the hot-hand portfolio also does not enter any of the optimal portfolios in Tables 1–4.

As Carhart (1997) points out, the hot-hand portfolio is a kind of momentum play. Even a strong belief in momentum, which in our setting amounts to a strong belief in Carhart's four-factor model, does not result in an allocation to the hot-hand strategy. As we discover, one reason for this outcome is the existence of other funds that apparently offer even stronger momentum plays, at least in the sense that they have higher coefficients on MOM. The first column of Table 3 displays the portfolio selected by an investor who rules out skill and has complete confidence in the four-factor model. Note that the bulk of this portfolio is invested in real estate funds. The regression results in Table 6 reveal that the posterior means of the MOM coefficients for many of these funds are higher than that for the hot-hand portfolio. Perhaps as importantly, the coefficients on SMB, HML, and MKT for these funds are also positive and relatively large. This is consistent with the evidence in Sanders (1997), who reports significantly positive SMB, HML, and MKT betas for real estate investment trust indices between 1978 and 1996. The highest-Sharpe-ratio portfolio of the benchmarks in the four-factor model contains those three factors and MOM in positive amounts. In our sample, real estate funds offer exposures to all four factors, and that feature makes them attractive to investors who believe in that model. When prior beliefs admit the possibility of skill, funds enter the optimal portfolio due to their average realized returns as well as their risk characteristics. This does not help the hot-hand portfolio, since the posterior mean of its δ_A is less than a basis point from zero.[11]

3.5. *What if the benchmarks were available for investment?*

In constructing the optimal portfolios analyzed in Tables 1–5, we preclude investment directly in the benchmark indexes, due to the fact that their returns omit any costs of implementing the underlying hypothetical investment strategies. The results discussed earlier reveal that perfect confidence in either the Fama-French or Carhart four-factor model does not result in an optimal portfolio of funds that closely mimics the optimal combination of the model's benchmark indexes. For an investor who has complete confidence in the Fama-French model and rules out skill ($\sigma_{\alpha_N} = \sigma_\delta = 0$), the correlation between the portfolio of funds and the optimal benchmark combination is only 0.75 (Table 2, Panel C). Moreover, such an investor judges the highest Sharpe ratio obtainable within the fund universe to be only 0.66 times that of the highest Sharpe ratio obtainable by combining the benchmarks.[12]

[11] The hot-hand porfolio has a positive alpha with respect to the Fama-French benchmarks and receives a substantial positive allocation when the investment universe contains only those three benchmarks and the hot-hand portfolio. Knox (1999) provides a treatment of this case in a Bayesian portfolio-choice setting.

[12] The latter number is not reported in the tables. If δ_0 were set to zero for each fund, the correlation between the two portfolios would equal the Sharpe ratio of the fund portfolio divided by the Sharpe ratio of the benchmark portfolio. In that case, the second portfolio would have the highest possible Sharpe ratio for the overall universe of funds and passive assets with investment weights unconstrained (i.e., short sales permitted), and an exact relation between correlations and Sharpe ratios applies (e.g., Kandel and Stambaugh, 1987; Shanken, 1987).

Under the Carhart model, the correlation between the optimal fund portfolio and the optimal combination of the four benchmarks is only 0.61 (Table 3, Panel C), and the Sharpe ratio of the first portfolio is only 0.54 times that of the second.[13]

Clearly, restricting the benchmark indexes to be unavailable for direct investment is not innocuous. To calibrate further the importance of that restriction, Table 7 compares the original funds-only portfolio with optimal portfolios computed under alternative scenarios that allow unrestricted long or short positions in one or more of the benchmarks. For example, the rows labeled "MKT" compare the original funds-only portfolio to the portfolio constructed from a universe that also allows long or short positions in that market index. As before, short fund positions are precluded. Recall that the fund universe offers close substitutes for a long position in MKT, so the principal difference here is the ability to short that index. Not surprisingly, for an investor who believes dogmatically in the CAPM and rules out skill, the ability to take a short position in MKT isn't valuable (worth only two basis points per month in certainty equivalent return with risk aversion again set to $A = 2.75$).

To an investor who centers his beliefs on the Fama-French model and rules out the possibility of skill, the ability to short MKT is worth a nontrivial 14 basis points per month (Panel B). Evidently, the ability to short MKT helps compensate to some degree for the inability otherwise to take the short positions inherent in SMB and HML. That compensation is only partial, however, since the ability to take positions directly in the latter two indexes is worth 66 basis points per month to that same investor, as indicated by the first entry in the second row of Panel B. (Recall that the optimal portfolio in the latter case has a correlation of only 0.75 with the original portfolio, as confirmed in Panel B.) To an investor who precludes skill and believes completely in the Carhart model, the ability to take positions directly in that model's four benchmarks is quite valuable – nearly 200 basis points per month beyond the value provided by the funds-only portfolio (Panel C).

The ability to take long or short positions in the benchmarks generally becomes more valuable as the investor admits some possibility of managerial skill. In essence, as the funds' track records lead to the inference that some of their δ_A's are positive, the ability to short MKT against such funds allows the investor to take large offsetting positions in the funds and MKT and thereby achieve high Sharpe ratios. As indicated by the results in Table 7, most of the enormous potential gains in such cases are indeed achieved by simply allowing short positions in MKT. That is, the increments to the certainty-equivalent return produced by allowing positions in the remaining benchmarks are relatively modest when σ_δ is 2% or more.

The main reason for the lack of benchmark substitutes is our precluding short sales of mutual funds. When the short-sale constraint is removed, the Sharpe ratio of the optimal fund portfolio increases to 0.99 times the Sharpe ratio of the efficient benchmark combination under the Fama-French model and to 0.94 times the

[13] The optimal combination of the Fama-French benchmarks is 32% in MKT, 4% in SMB, and 64% in HML, and the optimal combination of the four factors is 16% in MKT, 15% in SMB, 40% in HML, and 29% in MOM. Since the factors are constructed as long-short spreads, an x% weight in a given factor is interpreted as going x cents long and x cents short in the factor's legs for each \$1 invested in cash.

Table 7

Comparisons of portfolios of no-load funds with and without the benchmark indexes available for investment

Portfolios formed from an investment universe of 503 no-load equity mutual funds with at least three years of return history through December 1998 are compared to portfolios formed from a universe of the same 503 funds plus one or more passive benchmark indexes. The latter indexes have returns denoted by MKT, the excess return on the value-weighted stock market, SMB, the difference between returns on small and large stocks, HML, the difference between returns on high and low book-to-market stocks, and MOM, the difference between returns on stocks with high and low returns over the previous year (excluding the most recent month). The pricing models considered are the Capital Asset Pricing Model of Sharpe (1964) and Lintner (1965), the three-factor Fama-French (1993) model, and the four-factor model of Carhart (1997), which adds a momentum factor to the Fama-French model. All reported correlations and certainty-equivalent differences are computed using the predictive distribution formed under the prior mispricing uncertainty (σ_{α_N}) and skill uncertainty (σ_δ) in the column heading. The certainty-equivalent difference is computed with relative risk aversion of 2.75. Each row compares the funds-only portfolio to the portfolio that can contain the funds as well as the benchmarks indicated in the left-hand row heading. The benchmarks can enter with either long or short positions, whereas short positions in the funds are precluded throughout.

Mispricing uncertainty (σ_{α_N}) in percent per year:	0	0	0	0	0	2	2	2	2	2
Skill uncertainty (σ_δ) in percent per year:	0	1	2	3	∞	0	1	2	3	∞
Panel A. Pricing-model beliefs centered on the CAPM										
Certainty-equivalent difference (basis points per month)										
MKT	2	2	245	908	4,788	1	4	279	959	4,859
MKT, SMB, HML	2	5	298	997	4,937	8	14	322	1,036	5,039
MKT, SMB, HML, MOM	2	6	299	1,020	5,384	26	30	324	1,044	5,407
Correlation ($\times$100)										
MKT	100	97	39	25	17	99	94	39	25	16
MKT, SMB, HML	100	92	36	24	17	92	86	36	24	16
MKT, SMB, HML, MOM	100	91	36	24	16	80	75	36	24	16
Panel B. Pricing-model beliefs centered on the Fama-French three-factor model										
Certainty-equivalent difference (basis points per month)										
MKT	14	34	297	959	4,817	10	32	320	998	4,876
MKT, SMB, HML	66	66	361	1,062	5,022	61	65	377	1,091	5,095
MKT, SMB, HML, MOM	66	67	361	1,085	5,471	75	79	378	1,100	5,474
Correlation ($\times$100)										
MKT	88	79	44	29	17	92	81	42	28	17
MKT, SMB, HML	75	72	41	27	17	74	73	40	27	16
MKT, SMB, HML, MOM	75	72	41	27	16	73	71	40	26	16
Panel C. Pricing-model beliefs centered on the Carhart four-factor model										
Certainty-equivalent difference (basis points per month)										
MKT	9	56	428	1,161	5,108	10	55	425	1,157	5,100
MKT, SMB, HML	73	123	488	1,249	5,395	71	120	485	1,244	5,381
MKT, SMB, HML, MOM	197	202	515	1,250	5,651	194	200	513	1,245	5,631
Correlation ($\times$100)										
MKT	95	77	40	26	16	94	78	40	26	16
MKT, SMB, HML	73	64	37	25	15	74	65	38	25	15
MKT, SMB, HML, MOM	61	59	37	25	15	61	60	38	25	15

maximum under the Carhart model (with $\sigma_\alpha = \sigma_\delta = 0$ in both cases). Since only a relatively small subset of funds can be shorted in practice, precluding short sales in our fund universe seems reasonable. We also redid the analysis with an expanded investment universe of 919 funds that includes funds with load fees. The improvement from including the load funds is surprisingly small, despite the fact that we ignore their load fees. With complete belief in the four-factor model and skill precluded, the Sharpe ratio rises to only 0.55 times the maximum achievable using the benchmarks, as compared to a multiple of 0.54 in the original no-load setting. Under the Fama-French model, the Sharpe ratio rises so little that it rounds, as before, to only 0.66 times the maximum achievable with the benchmarks. Thus, the universe of all equity mutual funds with at least three years of history as of December 1998, including the load funds, provides no close substitutes for the Fama-French and momentum benchmarks.

A positive value for a fund's alpha, the intercept in Eq. (3), indicates that adding the fund to a universe containing only the benchmarks raises the maximum Sharpe ratio. Baks et al. (2001) explore the role of informative prior beliefs about alpha in forming an inference that the posterior mean of a fund's alpha is positive. Their fund-by-fund analysis addresses the question of whether any funds would be attractive to an investor who can already invest directly in the benchmarks, but it does not address the overall value to such an investor of the ability to select one or more mutual funds from the available universe. That value is computed in Table 8, which compares portfolios containing only benchmarks to portfolios containing both benchmarks and funds.

The results in Table 8 confirm, not surprisingly, that if all of a pricing model's benchmark indexes are available for investment, then mutual funds have no value to an investor who believes completely in the pricing model and rules out skill. If that investor maintains a belief in the model's ability to price passive assets but admits some possibility of managerial skill, then funds become valuable. When σ_δ is 2%, the ability to add funds to a portfolio containing only the model's benchmarks is worth at least 263 basis points per month to an investor who believes completely in any of the three pricing models. As in the comparisons presented in Table 7, the ability to take short positions in the benchmarks, especially MKT, makes the opportunity to take long fund positions quite valuable as skill uncertainty increases. The results in Table 8 also show that mispricing uncertainty has only minor effects on the value of having mutual funds available to an investor who can invest directly in the benchmarks. That is, to an investor who can earn the hypothetical costless returns on the benchmark indexes, the incremental value of mutual funds lies primarily in the potential skill of fund managers, as opposed to allowing the investor to exploit the inability of the benchmarks to price other passive assets.

3.6. How have the strategies performed?

The portfolios analyzed previously are selected using data through the end of our sample period, so the portfolios are optimal at that point in time under the various prior beliefs about pricing and skill. To investigate the effects that differences in

Table 8
Comparisons of portfolios of the benchmark indexes with and without the no-load funds available for investment

Portfolios containing one or more passive benchmark indexes are compared to portfolios that combine those indexes with any of the 503 no-load equity mutual funds with at least three years of return history through December 1998. The benchmark indexes have returns denoted by MKT, the excess return on the value-weighted stock market, SMB, the difference between returns on small and large stocks, HML, the difference between returns on high and low book-to-market stocks, and MOM, the difference between returns on stocks with high and low returns over the previous year (excluding the most recent month). The pricing models considered are the Capital Asset Pricing Model of Sharpe (1964) and Lintner (1965), the three-factor Fama-French (1993) model, and the four-factor model of Carhart (1997), which adds a momentum factor to the Fama-French model. All of the reported correlations and certainty-equivalent differences are computed using the predictive distribution formed under the prior mispricing uncertainty (σ_{α_N}) and skill uncertainty (σ_δ) in the column heading. The certainty-equivalent difference is computed with relative risk aversion equal to 2.75. Each row compares the portfolio containing only the benchmarks in the left-hand row heading to the portfolio that can contain the funds as well as those benchmarks. The benchmarks can enter with either long or short positions, whereas short positions in the funds are precluded throughout.

Mispricing uncertainty (σ_{α_N}) in percent per year:	0	0	0	0	0	2	2	2	2	2
Skill uncertainty (σ_δ) in percent per year:	0	1	2	3	∞	0	1	2	3	∞
Panel A. Pricing-model beliefs centered on the CAPM										
Certainty-equivalent difference (basis points per month)										
MKT	0	4	263	943	4,908	5	12	301	997	4,966
MKT, SMB, HML	0	7	316	1,032	5,056	1	11	333	1,064	5,135
MKT, SMB, HML, MOM	0	8	316	1,056	5,503	0	8	317	1,052	5,484
Correlation ($\times 100$)										
MKT	100	93	30	17	7	92	83	29	16	7
MKT, SMB, HML	100	89	28	16	7	99	88	32	19	9
MKT, SMB, HML, MOM	100	88	28	16	7	100	93	39	23	10
Panel B. Pricing-model beliefs centered on the Fama-French three-factor model										
Certainty-equivalent difference (basis points per month)										
MKT	38	65	342	1,019	4,936	39	66	363	1,054	4,989
MKT, SMB, HML	0	7	316	1,031	5,051	0	9	329	1,057	5,118
MKT, SMB, HML, MOM	0	8	316	1,055	5,500	0	8	316	1,052	5,481
Correlation ($\times 100$)										
MKT	64	54	27	16	7	63	54	26	16	7
MKT, SMB, HML	100	97	52	32	15	100	96	51	32	15
MKT, SMB, HML, MOM	100	97	52	32	14	100	97	54	33	15
Panel C. Pricing-model beliefs centered on the Carhart four-factor model										
Certainty-equivalent difference (basis points per month)										
MKT	64	114	480	1,218	5,212	68	115	480	1,215	5,203
MKT, SMB, HML	38	91	451	1,217	5,409	39	91	450	1,213	5,395
MKT, SMB, HML, MOM	0	8	316	1,055	5,503	0	8	316	1,051	5,483
Correlation ($\times 100$)										
MKT	54	43	23	15	7	53	43	23	15	7
MKT, SMB, HML	87	75	45	30	15	87	75	45	30	15
MKT, SMB, HML, MOM	100	99	68	46	22	100	99	68	46	22

prior beliefs can have on actual performance, we examine the out-of-sample returns on portfolios formed at earlier points in time, beginning 20 years before the end of our full sample. Since our methodology relies on long histories of passive asset returns, and these histories go back only to July 1963 in our sample, beginning the out-of-sample exercise earlier in the sample would introduce very noisy estimates of the passive-asset moments. Each month we construct ex ante-optimal portfolios by combining the different prior beliefs about skill and pricing with historical asset returns up through that month. The portfolios are rebalanced each month to incorporate the additional return history as well as changes in the fund universe, which consists of all no-load funds with at least three years of available data.

Table 9 reports the Sharpe ratios of the strategies for the last 20 years, January 1979 through December 1998, as well as for the first and second 10-year subperiods. For the 20-year period, the annualized ex post Sharpe ratios range from 0.42 to 0.66, depending on prior beliefs about pricing and skill. Not surprisingly, the differences across strategies are even more substantial within the shorter subperiods. Thus, not only do the various priors lead to important differences in ex ante performance (as in Table 5), they would have also produced some nontrivial differences ex post. For comparison, we also report the Sharpe ratios of the value-weighted market portfolio, the hot-hand strategy described in the previous section, and a "five-diamond" strategy. To implement the latter strategy, we sort all funds each month by their sample Sharpe ratios over the previous ten years (or less, if ten years of data are not available) and then equally weight the funds in the top decile – the five-diamond funds.[14] The Sharpe ratios of these three alternative strategies generally fall somewhere within the range of Sharpe ratios produced by the different beliefs about skill and pricing.

For a given belief about pricing, investors with no prejudice against managerial skill ($\sigma_\delta = \infty$) generally did somewhat better than investors who precluded it ($\sigma_\delta = 0$) over this particular sample period. The reader is cautioned from reading too much into such results, since 20 years is still a fairly short period over which to judge differences in ex post performance of equity strategies. In other words, such results are unlikely to tell any but the most indifferent investor the "correct" prior to use in going forward. (Note, for example, that an investor who picked his strategy using ex post performance through 1988 would not have experienced the highest subsequent performance.) Based on the 20-year performance, believing dogmatically in the CAPM and ruling out skill would seem as good a set of beliefs as any, but clearly many investors with other views about pricing and skill would not be so easily deterred. Our intention here is simply to provide some additional historical perspective on the important role of prior beliefs in the mutual-fund investment decision.

[14] This ranking is similar in spirit to the five-star ranking by Morningstar, Inc., a leading provider of mutual fund information. Morningstar ranks funds into five categories (one to five stars) based on a risk-adjusted rating in which a measure of the fund's downside volatility is subtracted from a measure of the fund's average excess return. Although the Sharpe ratio and the Morningstar rating are defined differently, they share the same basic risk-adjustment concept and often provide similar rankings of funds, as demonstrated by Sharpe (1997, 1998).

Table 9
Out-of-sample performance of various investment strategies
Sample Sharpe ratios are computed for investment strategies corresponding to various prior beliefs about pricing and skill. Prior mispricing uncertainty, σ_α, corresponds to the pricing model given in the same row. Prior skill uncertainty (σ_δ) as well as σ_α are reported in percent per year. All strategies are rebalanced monthly and rely only on information available up to that month. The investment universe in any given month consists of all no-load equity mutual funds with at least three years of return history. Every month, the funds are sorted according to their sample Sharpe ratios over the last ten years (or less, if ten years of data are not available), and the funds in the top decile are assigned "five diamonds". The five-diamond strategy buys an equally-weighted portfolio of all five-diamond funds. The hot-hand strategy buys an equally-weighted portfolio of the top decile of funds ranked on their returns over the previous calendar year.

	Sample Sharpe ratio (annual)		
Investment strategy	Jan 79–Dec 98	Jan 79–Dec 88	Jan 89–Dec 98
CAPM, $\sigma_\alpha = 0$, $\sigma_\delta = 0$	0.66	0.31	0.93
CAPM, $\sigma_\alpha = 0$, $\sigma_\delta = 2$	0.51	0.07	0.87
CAPM, $\sigma_\alpha = 0$, $\sigma_\delta = \infty$	0.66	0.53	0.77
CAPM, $\sigma_\alpha = 2$, $\sigma_\delta = 0$	0.52	0.30	0.68
CAPM, $\sigma_\alpha = 2$, $\sigma_\delta = 2$	0.53	0.18	0.82
CAPM, $\sigma_\alpha = 2$, $\sigma_\delta = \infty$	0.64	0.54	0.73
Fama-French, $\sigma_\alpha = 0$, $\sigma_\delta = 0$	0.46	0.25	0.69
Fama-French, $\sigma_\alpha = 0$, $\sigma_\delta = 2$	0.56	0.28	0.81
Fama-French, $\sigma_\alpha = 0$, $\sigma_\delta = \infty$	0.66	0.57	0.74
Fama-French, $\sigma_\alpha = 2$, $\sigma_\delta = 0$	0.48	0.25	0.69
Fama-French, $\sigma_\alpha = 2$, $\sigma_\delta = 2$	0.54	0.31	0.75
Fama-French, $\sigma_\alpha = 2$, $\sigma_\delta = \infty$	0.65	0.56	0.72
4-factor, $\sigma_\alpha = 0$, $\sigma_\delta = 0$	0.46	0.44	0.48
4-factor, $\sigma_\alpha = 0$, $\sigma_\delta = 2$	0.45	0.38	0.52
4-factor, $\sigma_\alpha = 0$, $\sigma_\delta = \infty$	0.61	0.54	0.67
4-factor, $\sigma_\alpha = 2$, $\sigma_\delta = 0$	0.48	0.43	0.53
4-factor, $\sigma_\alpha = 2$, $\sigma_\delta = 2$	0.44	0.38	0.50
4-factor, $\sigma_\alpha = 2$, $\sigma_\delta = \infty$	0.60	0.53	0.66
No model, $\sigma_\alpha = \infty$, $\sigma_\delta = 0$	0.48	0.38	0.57
No model, $\sigma_\alpha = \infty$, $\sigma_\delta = 2$	0.42	0.35	0.48
No model, $\sigma_\alpha = \infty$, $\sigma_\delta = \infty$	0.58	0.52	0.63
Value-weighted market portfolio	0.65	0.45	0.91
Five-diamond strategy	0.66	0.60	0.77
Hot-hand strategy	0.63	0.50	0.80

4. Conclusions

This study develops and applies a framework in which beliefs about pricing models and managerial skill are combined with information in the data to select portfolios of mutual funds. Nonbenchmark passive assets provide additional information about the mutual funds' expected returns, and they allow us to specify prior beliefs that distinguish mispricing from skill. In addition, nonbenchmark assets help account for common variation in fund returns, making the investment problem feasible with a large universe of funds.

We construct portfolios with maximum Sharpe ratios from a universe of 503 no-load equity mutual funds. The optimal portfolios are substantially affected by prior beliefs about pricing and skill as well as by including the information in nonbenchmark assets. A pricing model is useful to an investor seeking a high Sharpe ratio, even if the investor has less than complete confidence in the model's pricing accuracy and cannot invest directly in the benchmarks. With investment in the benchmarks precluded, even investors who believe completely in a pricing model and rule out the possibility of manager skill can include active funds in their portfolios. The fund universe offers no close substitutes for the Fama-French and momentum benchmarks, and active funds can be better substitutes for the benchmarks than passive funds. We also find that the hot-hand portfolio of the previous year's best-performing funds does not appear in the portfolio of funds with the highest Sharpe ratio, even when momentum is believed to be priced.

Maximizing the Sharpe ratio is only one of many investment objectives. With a multiperiod investment objective, for example, beliefs about pricing and skill could exhibit different effects. A multiperiod setting could also allow a meaningful consideration of the funds that charge load fees. Incorporating changes over time in fund betas would also be desirable. Such extensions offer challenges for future research.

Appendix

This appendix derives the moments of the predictive distribution of the fund returns. We first provide the predictive moments of the returns on passive assets. Those moments are then combined with the posterior moments of the parameters in Eqs. (1) and (2), derived in Pástor and Stambaugh (2002), to obtain the predictive moments of the fund returns.

Define $Y = (r_{N,1},\ldots,r_{N,T})'$, $X = (r_{B,1},\ldots,r_{B,T})'$, and $Z = (\iota_T \;\; X)$, where ι_T denotes a T-vector of ones. Also define the $(k+1)\times m$ matrix $G = (\alpha_N \;\; B_N)'$, and let $g = vec(G)$. For the T observations $t = 1,\ldots,T$, the regression model in Eq. (1) can be written as

$$Y = ZG + U, \quad vec(U) \sim \mathrm{N}(0, \Sigma \otimes I_T), \tag{A.1}$$

where $U = (\varepsilon_{N,1},\ldots,\varepsilon_{N,T})'$. Let E_B and V_{BB} denote the mean and covariance matrix of the normal distribution for $r_{B,t}$, let θ_P denote the parameters of the joint distribution of the passive asset returns (G, Σ, E_B, and V_{BB}), and define the $T\times p$ sample matrix of passive returns, $R_P = (X \;\; Y)$. The appendix of Pástor and Stambaugh (2002) reports the posterior moments of the elements of θ_P. Those moments include the posterior mean and variance of g, denoted by $\tilde{g}$ and $Var(g|R_P)$, the posterior mean and variance of E_B, denoted by $\tilde{E}_B$ and $Var(E_B|R_P)$, and the posterior means of Σ and V_{BB}, denoted by $\tilde{\Sigma}$ and $\tilde{V}_{BB}$. Posterior means are denoted using tildes throughout the appendix.

The predictive moments of the passive returns are derived in Pástor and Stambaugh (2000) in a different context. Define $r_{P,T+1} = (r'_{N,T+1} r'_{B,T+1})'$. Its predictive mean is

$$E_P^* = \mathrm{E}(r_{P,T+1}|R_P) = \begin{pmatrix} \tilde{\alpha}_N + \tilde{B}_N \tilde{E}_B \\ \tilde{E}_B \end{pmatrix}, \tag{A.2}$$

where $\tilde{\alpha}_N$ and $\tilde{B}_N$ are obtained using $\tilde{g} = vec((\tilde{\alpha}_N \;\; \tilde{B}_N)')$. Partition the predictive covariance matrix as

$$V_P^* = Var(r_{P,T+1}|R_P) = \begin{bmatrix} V_{NN}^* & V_{NB}^* \\ V_{BN}^* & V_{BB}^* \end{bmatrix}. \tag{A.3}$$

Denote the ith row of B_N as b_i', the ith column of G as g_i, and the (i,j) element of Σ as $\sigma_{i,j}$. The first submatrix, V_{NN}^*, can be represented in terms of its (i,j) element:

$$\begin{aligned}(V_{NN}^*)_{(i,j)} &= \tilde{b}_i' V_{BB}^* \tilde{b}_j + tr[V_{BB}^* \; Cov(b_i, b_j'|R_P)] + \tilde{\sigma}_{i,j} \\ &\quad + [1 \;\; \tilde{E}_B'] Cov(g_i, g_j'|R_P)[1 \;\; \tilde{E}_B']'. \end{aligned} \tag{A.4}$$

Note that $Cov(b_i, b_j'|R_P)$ and $Cov(g_i, g_j'|R_P)$ are submatrices of $Var(g|R_P)$. The remaining submatrices in Eq. (A.3) can be shown to be equal to

$$V_{BB}^* = \tilde{V}_{BB} + Var(E_B|R_P),$$
$$V_{NB}^* = V_{BN}^{*'} = \tilde{B}_N \tilde{V}_{BB} + \tilde{B}_N Var(E_B|R_P).$$

Let us now turn to the regression model in Eq. (2), which can be written as

$$r_{A,T+1} = \delta_A + c_A' r_{P,T+1} + u_{T+1} \tag{A.5}$$

$$= [1 \;\; r'_{P,T+1}]\phi_A + u_{T+1}, \tag{A.6}$$

where $\phi_A = (\delta_A \;\; c_A')'$. Let R denote all of the sample returns data on funds and passive assets through period T, and let θ_A denote the set of parameters ϕ_A and σ_u^2. The posterior moments of the elements of θ_A are reported in the appendix of Pástor and Stambaugh (2002). Those moments include the posterior mean and variance of ϕ_A, denoted by $\tilde{\phi}_A$ and $Var(\phi_A|R)$, and the posterior mean of σ_u^2, denoted by $\tilde{\sigma}_u^2$.

The derivation of the predictive moments of fund returns parallels the derivation in Pástor and Stambaugh (2000) of the predictive moments of the nonbenchmark returns, $r_{N,T+1}$. Since c_A and E_P (the mean of $r_{P,t}$) are independent in the prior, the predictive mean of $r_{A,T+1}$ is

$$\mathrm{E}(r_{A,T+1}|R) = \mathrm{E}(\delta_A + c_A' E_P|R) = \tilde{\delta}_A + \tilde{c}_A' \tilde{E}_P. \tag{A.7}$$

Note that $\tilde{E}_P$, the posterior mean of E_P, is equal to the predictive mean E_P^*. The predictive variance of $r_{A,T+1}$ can be written as

$$Var(r_{A,T+1}|R) = \mathrm{E}(Var(r_{A,T+1}|R, \phi_A)|R) + Var(\mathrm{E}(r_{A,T+1}|R, \phi_A)|R). \tag{A.8}$$

To compute the first term on the right-hand side of Eq. (A.8), observe using Eq. (A.5) that

$$Var(r_{A,T+1}|R, \phi_A) = c_A' V_P^* c_A + \tilde{\sigma}_u^2, \tag{A.9}$$

since the predictive variance of u_{T+1} equals the posterior mean of σ_u^2 by the law of iterated expectations (conditioning on σ_u^2). Taking expectations gives

$$\mathrm{E}(Var(r_{A,T+1}|R, \phi_A)|R) = \tilde{c}_A' V_P^* \tilde{c}_A + tr[V_P^* \; Cov(c_A, c_A'|R)] + \tilde{\sigma}_u^2. \tag{A.10}$$

To compute the second term on the right-hand side of Eq. (A.8), observe using Eq. (A.6) that

$$\mathrm{E}(r_{A,T+1}|R, \phi_A) = [1 \;\; \tilde{E}_P'] \phi_A, \tag{A.11}$$

so

$$Var(\mathrm{E}(r_{A,T+1}|R, \phi_A)|R) = [1 \;\; \tilde{E}_P'] Cov(\phi_A, \phi_A'|R) [1 \;\; \tilde{E}_P']'. \tag{A.12}$$

Note that $Cov(c_A, c_A'|R)$ is a submatrix of the posterior covariance matrix $Var(\phi_A|R) \equiv Cov(\phi_A, \phi_A'|R)$.

Computing the predictive covariance of $r_{A,T+1}$ with the return on another fund J, $r_{J,T+1}$, is simplified by the independence across funds of (i) the disturbances in Eq. (A.5) and (ii) the posteriors for the coefficient vectors ϕ_A and ϕ_J. Applying the same approach as used above for the predictive variance gives

$$Cov(r_{A,T+1}, r_{J,T+1}|R) = \tilde{c}_A' V_P^* \tilde{c}_J. \tag{A.13}$$

Computing the predictive covariance of $r_{A,T+1}$ with the vector of returns on the passive assets, $r_{P,T+1}$, is simplified by the independence of the posterior for ϕ_A from that of E_P and V_P. Let θ denote the union of θ_P and θ_A. Using the law of iterated expectations and the variance decomposition rule gives

$$\begin{aligned} Cov(r_{A,T+1}, r_{P,T+1}|R) &= \mathrm{E}(Cov(r_{A,T+1}, r_{P,T+1}|R, \theta)|R) \\ &\quad + Cov(\mathrm{E}(r_{A,T+1}|R, \theta), \mathrm{E}(r_{P,T+1}|R, \theta)|R) \\ &= \mathrm{E}(V_P c_A|R) + Cov(\delta_A + c_A' E_P, E_P|R) \\ &= \tilde{V}_P \tilde{c}_A + Cov(E_P, E_P'|R) \tilde{c}_A \\ &= V_P^* \tilde{c}_A. \end{aligned} \tag{A.14}$$

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