Mutual fund tournament:
Risk taking incentives induced by ranking objectives[*]

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Abstract

Mutual funds' performance rankings are widely publicized in the media and have a strong impact on fund flows (see, e.g., Sirri and Tufano, 1998). In this paper, we study risk taking incentives of mutual fund managers who have ranking objectives (as in a tournament). First, in a two-period model, we analyze the game played by two risk-neutral fund managers with ranking objectives. We show that in the first period, managers choose the same risk level but do not act in the interest of investors. In the second period the interim loser *(i)* takes more risk than the interim winner and *(ii)* the level of risk undertaken by the interim loser is increasing with the difference in interim performances. Second, we empirically test some predictions of the model in a sample of diversified US equity funds in 1976-1999, using a more powerful methodology than in previous studies and accounting for cross-correlation in fund returns. We find evidence that funds' choice of systematic risk in the second part of the year is negatively related to their interim performance, which is consistent with the model.

Keywords: ranking-based objectives, interim performance, risk-taking incentives.
JEL Classification: G11, G24.

1   Introduction

When choosing between mutual funds, investors take into account many considerations such as fund performance, reputation, fee structure, the diversity and size of the fund's family, etc. (see, e.g., Chevalier and Ellison, 1997, and Sirri and Tufano, 1998). Naturally, a rational investor will select a fund, which offers the best combination of the relevant factors. Since fund performance seems to be the most important selection criterion for consumers (see Capon, Fitzsimons, and Prince, 1996), they typically choose funds that have high raw or risk-adjusted performance relative to their peer group (see, e.g., Ippolito, 1992, Chevalier and Ellison, 1997, Lettau, 1997, and Sirri and Tufano, 1998). The information about fund performance rankings is regularly published in the financial media (the Wallstreet Journal, Business Week, Money, etc.) and is often referred to in funds' advertisements. The importance of rankings in describing fund performance is illustrated by Gould (1998):

> *"Bartlett Europe has returned an annual average of* 27.2 *percent for the three years through Dec. 4, ranking first among the 46 European stock funds tracked by Morningstar Inc."*

The academic literature also points out the importance of fund performance rankings for investors, documenting that rankings may have higher impact on fund flows than returns (see, e.g., Patel, Zeckhauser and Hendricks, 1994, Massa, 1997, and Goriaev, 2003). Such investors' behavior induces ranking-based objectives for fund managers, since their compensation is typically based on a percentage of the fund's assets (see Khorana (1996) and Deli (2002)[1]).

The goal of this paper is to investigate how ranking objectives influence managers' investment strategies, and test empirically some predictions of the model. In the first part of the paper, we develop a model in which, during two investment periods, two risk-neutral managers compete for future money flows and observe their interim relative performance. We show that in the first period, managers choose the same risk level but do not maximize their expected return. In the second period, the interim loser *(i)* increases risk with respect to the first period while the interim winner decreases risk, *(ii)* the difference in risk undertaken is increasing with the difference in interim performances and *(iii)* the interim loser may act more in the interest of investors (i.e., choose a strategy with a higher expected return) than the interim winner.[2]

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[1]Khorana mentions that "the investment advisor's compensation is directly linked to the fund's size; the advisor receives a management fee based on a percentage of average net assets held during the year." In a sample of 5, 198 funds, Deli (2002) finds that 4, 833 funds had advisory contracts based solely on a percentage of assets.

[2]Using simulations, we also provide evidence that manager's choice of risk in the second period is negatively related to his relative performance in the first period in the case with more than two competing funds.

In the second part of the paper, we apply a new methodology to empirically test some predictions of the model in a sample of diversified US equity funds in 1976-1999. In contrast to the previous studies (see, e.g., Brown, Harlow and Starks, 1996, and Koski and Pontiff, 1999), our statistical tests take into account the presence of the cross-correlation in fund returns highlighted by Busse (2001). We find evidence that funds' choice of systematic risk in the second part of the year is negatively related to their category-relative and class-relative interim performance, which is consistent with the model.

The organization of this paper is as follows. Section 2 reviews the related literature, Section 3 presents the model, Section 4 derives the equilibrium, Section 5 considers the case with several competing funds, Section 6 presents the empirical results, and Section 7 concludes.

2   Related literature

A growing body of literature studies the mutual fund tournament both theoretically and empirically. Closely related theoretical papers studying relative performance evaluation in financial markets are those of Huddart (1999), Hvide (2002), Palomino (2002), and Taylor (2003). In this type of the models, a manager's payoff depends not only on his own strategy, but also on the other managers' strategies. In this respect, these models and our model are different from those analyzing the behavior of a manager evaluated against an exogenous benchmark (see, e.g., Grinblatt and Titman, 1989, Admati and Pfleiderer, 1996, Carpenter, 2000).

Hvide (2002) and Palomino (2002) study the consequences of relative performance objective in the context of a single investment decision. Hvide shows that in a situation with moral hazard on both effort and risk, standard tournament rewards induce excessive risk and lack of effort. Palomino (2002) assumes that managers with different levels of information compete in oligopolistic markets and aim at maximizing their relative performance against the average performance in their category. He shows that despite the objective function being linear in performances, managers have incentives to choose overly-risky strategies. Furthermore, relative performance objectives always lead to under-acquisition of information. Huddart (1999) considers a two-period model in which interim performances are observable. He shows that asset-based compensation schemes generate incentives for managers to invest in overly-risky portfolio in the first period, and that performance fees align managers' incentives with those of investors.

Our theoretical results should be compared with those of Cabral (1997) on the choice of R&D projects. Cabral considers an infinite-period race between two firms that choose between low variance projects (low gains with high probability) and high variance projects

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(large gains with low probability). If the two firms choose a project of the same type then outcomes are perfectly correlated. Cabral shows that in equilibrium, both firms choose overly risky R&D strategies. There are three main differences between Cabral's model and ours. First, in Cabral's model, players have an infinite horizon. It follows that strategy choices are not influenced by an "end of the game" effect. Second, players receive a payoff in every period. This is equivalent to assuming observable interim performance. Conversely, in our model, players face an end of the game and only receive a payoff at the end of the game. Finally, in Cabral's R&D race, projects' payoffs are different only in case of success. If projects fail, the costs faced by firms are independent of the projects chosen. This implies that an intermediate loser only catches up with the leader if a good outcome is realized. The situation is different in the mutual fund tournament. An intermediate loser has two ways of catching up with the winner: by winning more in case of good outcomes or by losing less is case of bad outcomes.

The consequences of dynamic incentives and relative performance evaluation have also been studied by Meyer and Vickers (1997). They show that in a dynamic principal-agent relationship, relative performance evaluation can be either welfare increasing or decreasing. The reason is that in a dynamic setting, there may be both explicit and implicit incentives and better information may decrease implicit incentives. Our model is different from that of Meyer and Vickers in two ways. First, in their model, intermediate performance is observable. Second, in our model, portfolio decisions are costless, i.e., they do not require any effort from fund managers. This is different from standard principal-agent models in which agents' output results from an costly effort.

Another strand of the literature conducts empirical analysis of fund managers' strategic behavior, focusing on the impact of past performance on funds' risk taking decisions. Several studies test the so-called tournament hypothesis that funds underperforming after the first part of the year increase risk in the second part of the year, trying to catch up with interim winners at the end of the year. Applying a contingency table methodology to the sample of US growth funds in 1976-1991, Brown, Harlow, and Starks (1996) find that interim losers (defined as funds below the median category return over the first part of the year) increase risk towards the end of the year relative to interim winners. Using a sample of US domestic equity funds in 1992-1994, Koski and Pontiff (1999) apply regression methodology and find a negative relationship between fund return over the first semester and the change in total, systematic, and unsystematic risk between the first and second semesters. Chevalier and Ellison (1997) use a different approach, measuring fund risk on the basis of the fund's portfolio holdings. They also find a negative relationship between fund return over the first nine months of the year and the change in fund risk between September and December, using a sample of growth and growth-and-income funds in 1982-1992. However, Busse

(2001) argues that these results should be taken with caution. He finds no evidence in favor of the tournament hypothesis, applying either the contingency table or the regression methodology to daily returns of US domestic equity funds in 1985-1995. He explains this divergence in the results by the presence of the auto- and cross-correlation in fund returns, which was not accounted for in the standard statistical tests used in the previous studies.

3 Presentation of the model

There are two periods, 1 and 2, and two risk-neutral money managers. At the beginning of the first period, each manager has $A \geq 0$ units of money under management. At the beginning of each period, managers choose an investment strategy. There is a continuum of investment strategies and the return of each strategy is log-normally distributed. The log-return

of a strategy is normally distributed with variance v and mean $m(v)$. Following Palomino and Prat (2003), we assume that the function $m(\cdot)$ is positive, twice differentiable, strictly concave with $m'''(\cdot) \geq 0$, and has a maximum at $\hat{m} = m(\hat{v})$ with $\hat{v}$ strictly positive.

A possible interpretation for the shape of $m(\cdot)$ is that there is no borrowing constraint but borrowing is increasingly costly. Therefore, there is a borrowing threshold beyond which the marginal borrowing cost exceeds the marginal expected return of investment.

Information about realized returns. After returns are realized at the end of period 1, managers observe both their performance and the performance of their opponent.

Managers' objective: Managers aim at maximizing the size of the funds under management at the end of period 2. The fund size can be increased in two ways. First, by realizing a high cumulated return over the periods 1 and 2, and second by attracting new funds.

There is a continuum of identical atomistic individual investors. On aggregate, these investors will have an amount of money $B > 0$ to invest at the end of period 2. Following empirical evidence provided by Patel, Zeckhauser and Hendricks (1994) and Massa (1997), we assume that investors put their money in the fund that has realized the higher cumulative return over periods 1 and 2. If funds perform equally well, each fund will get an amount $B/2$.

Under such an assumption, the objective of manager i is to maximize

$$C_i = \begin{cases} AR_{i,1}R_{i,2} + B & \text{if} \quad R_{i,1}R_{i,2} > R_{j,1}R_{j,2} \\ AR_{i,1}R_{i,2} + B/2 & \text{if} \quad R_{i,1}R_{i,2} = R_{j,1}R_{j,2} \\ AR_{i,1}R_{i,2} & \text{if} \quad R_{i,1}R_{i,2} < R_{j,1}R_{j,2} \end{cases} \tag{1}$$

with $j \neq i$, and where $R_{i,t}$ represents the gross return realized by manager i in period t.

Our model captures the following idea in a simple framework. First, investors use rankings as a rule of thumb to evaluate managers and allocate capital to funds (as empirical evidence provided by Patel, Zeckhauser and Hendricks (1994) and Massa (1997) suggests). Second, fund managers are risk-neutral agents who maximize the size of the fund they manage.

Before proceeding, several remarks should be made. First, we do not question whether fund investors are right or wrong to use rankings as a rule of thumb to evaluate fund managers. Rather, we study the *consequences* of this observed behavior.

Second, following Das and Sundaram (2002) and Palomino and Uhlig (2002), we depart from the traditional principal-agent approach to contracting in which the principal (i.e., the investor) decides the compensation contract of the agent (i.e., the fund manager). In the mutual fund industry, funds (i.e., agents) choose the type of fee they charge to investors (principals). In our model, the compensation scheme (i.e., an asset-based compensation) is given. As evidenced by Deli (2002), this compensation scheme is used by more than 90% of funds.

Third, it is assumed that portfolios are unobservable. We believe that this assumption is realistic, since portfolio disclosures are not frequent[3] and managers window-dress their portfolio around disclosure dates in practice (see, e.g., Musto, 1999, and Carhart et al., 2002).

Also, we assume that returns realized by managers are uncorrelated. This implies that the only strategic decision of the managers is the variance of their portfolio. A more complete model would assume that a manager can also influence the covariance of returns. Such a case is considered in Appendix B. It is shown that there exist equilibria such that the results about risk taking incentives generated by ranking objectives derived in the case of uncorrelated returns still hold, qualitatively, in the case of correlated returns.

Finally, it can be argued that investors evaluate managers with respect to each other only if the two managers are of different qualities. This may not be the case. It is sufficient that investors *believe* that managers are of different qualities. For example, consider the following situation. With probability $1/2$, manager i is a high quality manager and with probability $1/2$ he is a bad quality manager, and probabilities of being a good manager are independent across managers. Moreover, the two managers observe the realized types while investors do not. In such a situation, with probability $1/2$, it is common knowledge among managers that they are of the same type. However, investors do not know whether managers are of the same type. According to investors' beliefs, with probability $1/2$, there is a good and a bad manager, and they uses a relative performance rule to evaluate managers.

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[3]In the United States, mutual fund portfolios have to be disclosed semiannually. However, other countries such as the Netherlands only require disclosure once a year.

Here, in order to concentrate on incentives generated by differences in intermediate performances, we solely study the case in which managers are of the same quality. If managers were of different qualities, incentives in period 2 would be driven by both interim performances and difference in quality.

The benchmark case

We consider as a benchmark the case in which managers maximize their expected return (i.e., $A > 0$ and $B = 0$). In such a situation, in each period, the expected return of a manager is $m(v) + v/2$. Hence, both managers choose a risk level $v = \bar{v}$ such that $m'(\bar{v}) = -\frac{1}{2}$. The goal of our model is to show how ranking objectives alter the managers' investment strategies.

Also, note that the same result holds if managers aim at maximizing the relative return, i.e., the difference between their return and that of their competitor, over the two periods. The reason is that in such a case, the objective function of the manager is linear in his return. As a consequence, maximization of own performance and maximization of relative performance lead to the same investment strategy.

4 Equilibrium investment strategies

We solve the model using backward induction. Hence, we start by deriving the equilibrium of the game played by the two managers in period 2. Denote $R_{t,w}$ and $R_{t,l}$ the gross return obtained in period by t by the interim winner and loser, respectively. Let $r_{j,t} = \log(R_{j,t})$ ($j = l, w$ and $t = 1, 2$) and $\delta = r_{w,2} - r_{l,2}$. From the assumption about the distribution of returns, $r_{l,2} - r_{w,2}$ is normally distributed with mean $m(v_l) - m(v_w)$ and variance $v_l + v_w$. Hence, the objective of the interim loser is to maximize

$$H_l(v_l, v_w, \delta) = A \exp\left(m(v_l) + \frac{1}{2}v_l\right) + B\left[1 - \Phi\left(\frac{\delta + m(v_w) - m(v_l)}{(v_l + v_w)^{1/2}}\right)\right] \tag{2}$$

over v_l, while the objective of the interim winner is to maximize

$$H_w(v_w, v_l, \delta) = A \exp\left(m(v_w) + \frac{1}{2}v_w\right) + B\Phi\left(\frac{\delta + m(v_w) - m(v_l)}{(v_l + v_w)^{1/2}}\right) \tag{3}$$

over v_w, where Φ is cdf of the standard normal distribution.

A manager's objective is to maximize the size of his fund at the end of the second period. This can be achieved in two ways. First, by obtaining a high return. This is captured by the first term in the right-hand sides of (2) and (3). This provides managers with incentives to maximize their expected return (i.e., choose $v = \bar{v}$). The second way of increasing the

size of the fund is by outperforming the opponent. This is captured by the second term in the right-hand sides of (2) and (3). The larger the ratio A/B, the more managers' incentives are aligned with investors' interests (i.e., the maximization of expected returns). Conversely, when the ratio A/B is small, managers' main objective is to outperform their opponent in order to receive B. To isolate incentives generated by tournament objectives, we concentrate on the case in which A is negligible with respect to B. (Technically, we assume that $A = 0$.) In such a situation, managers' only objective is to be ranked first. We have the following results.

Proposition 1 *Assume that managers' objective function is given by (1) with $A = 0$.*
(i) If $\delta \neq 0$, then in the second period, the unique equilibrium is such that $v_w^* < \hat{v} < v_l^*$ with $|m'(v_w^*)| = |m'(v_l^*)|$. Furthermore, v_l^* and v_w^* are increasing and decreasing in δ, respectively.
(ii) If $\delta = 0$, then there exists a unique symmetric equilibrium in the second period: both managers choose $\hat{v}$.

Proof: See Appendix A.

Proposition 1 states that if the two funds have performed differently in the first period ($\delta \neq 0$) then, in the last period, the unique equilibrium is such that an interim loser takes more risk than an interim winner. Furthermore, the larger the difference in performance between the interim winner and the interim loser at the end of period 1, the larger the difference in risk undertaken in period 2. If managers have performed equally well in the first period ($\delta = 0$), they both choose a conservative strategy ($\hat{v}$) in the second period. The reason for this last result is that if manager i chooses $\hat{v}$ and manager $j \neq i$ does not, then manager i has a probability of winning the contest strictly larger than $1/2$, while if manager j chooses $\hat{v}$, both managers have a probability $1/2$ of winning the contest. Conversely, $\bar{v}$ (i.e., the risk level maximizing the expected return) is never a best reply to $\bar{v}$, the reason being that the distribution of returns is not symmetric around its mean.

By the same argument, we derive equilibrium strategies played in the first period.

Proposition 2 *Assume that managers' objective function is given by (1) with $A = 0$.* *There exists a unique symmetric equilibrium in the first period: both managers choose $\hat{v}$.*

Proof: See Appendix A.

Proposition 2 implies that in the first period managers do not act in the interest of investors. They choose a risk level smaller than $\bar{v}$. As already mentioned, the reason is that the log-normal distribution is not symmetric with respect to its mean. It follows that if one manager chooses $v = \bar{v}$, then the best reply of his opponent is not $\bar{v}$.

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From Propositions 1 and 2, we deduce that when compensation is exclusively based on ranking, an interim winner locks in his gain in the second period, hence decreasing his level of risk undertaken with respect to the first period. Conversely, the interim loser increases risk with respect to the first period. Note, however, that if δ is small, we have $v_w^* < \hat{v} < v_l^* < \bar{v}$. This implies that in the second period the interim loser acts more in the interest of investors than interim winners.

5   More than two competing funds

So far, we have assumed that there are only two competing funds. In this section, we consider a more realistic case in which there are more than two competing funds. Denote $N > 2$ the number of competing funds and assume that fund i receives the investors' money if it has the highest return over two periods:

$$\begin{array}{ll} C_i = B & \text{if } R_{i,1}R_{i,2} > R_{j,1}R_{j,2} \ (i \neq j) \\ C_i = 0 & \text{otherwise} \end{array} \qquad (4)$$

Let $\delta_{ij} = r_{i,1} - r_{j,1}$. The objective of fund i in period 2 is to maximize

$$\text{Prob}\left(r_{i,2} > \max_{j \neq i}(r_{j,2} - \delta_{ij})\right).$$

If fund returns are uncorrelated, this is equivalent to maximizing

$$\prod_{j \neq i} \text{Prob}\left(r_{2,i} > r_{2,j} - \delta_{ij}\right).$$

Let

$$G(v_i, v_j, \delta_{ij}) = \frac{\delta_{ij} + m(v_i) - m(v_j)}{(v_i + v_j)^{1/2}}.$$

Given that log-returns are normally distributed, the first-order condition of the maximization program of manager i $(i = 1, \ldots, N)$ is

$$B \sum_{j \neq i} \frac{\partial G}{\partial v_i}(v_i^*, v_j^*, \delta_{ij})f[G(v_i^*, v_j^*, \delta_{ij})] \left\{\Pi_{k \neq j,i}[1 - F(G(v_i^*, v_k^*, \delta_{ik}))]\right\} = 0. \qquad (5)$$

To derive some analytical results is quite a difficult task. Therefore, we rely on numerical simulations to provide evidence that the results of Proposition 1 hold in the case with more than two competing funds. In our basic simulations, we assume that $m(v) = 1 - (1 - v)^2$ and $N = 3$. In such a case $\hat{v} = 1$ and $\bar{v} = 5/4$. We denote v_w, v_s and v_l the risk levels undertaken in the second period by the funds ranked first, second and third at the end of

the first period, respectively. We obtain the following results for 100 observations[4]. For each observation, we have $v_w < v_s < v_l$, $v_w < \hat{v}$ and $v_l > \hat{v}$. This means that *(i)* the risk level chosen in the second period is negatively correlated with the interim performance and *(ii)* the interim winner (w) always decreased his risk level in the second period, while the interim loser (l) always increased his risk level. The fund ranked second either increased or decreased its risk level depending on the performance of the two other funds. Aggregate results from the simulations are given in the following table.

	v_w	v_s	v_l
Average	0.860	1.070	1.390
Std Dev	0.066	0.065	0.668
Max	0.950	1.193	3.501
Min	0.715	0.985	1.045

We observe that, on average, the fund ranked second increased its risk level in the second period.

A final observation is that, on average, the interim loser acts more in the interest of investors that other funds. Its average risk level in the second period (1.39) is the closest to the risk level maximizing expected return (1.25). This confirms the remarks made about the results of Proposition 1 that the interim loser may act more in the interest of investors than the interim winner.

6 Empirical evidence

6.1 Data

Our data come from the CRSP Survivor-Bias Free Mutual Fund Database[5]. The sample includes diversified US equity funds[6] that took part in twenty four annual tournaments from 1976 to 1999. The data relevant for our analysis include fund starting dates, monthly returns, quarterly TNA and, starting from 1992, annual Wiesenberger, ICDI, and Strategic

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[4]For each observation, three independent returns of a random variable normally distributed with mean $m(\bar{v})$ and variance $\bar{v}$ are generated. Then, the system of equations (5) is solved numerically under the second-order-condition constraints.

[5]Source: CRSP, Center for Research in Security Prices. Graduate School of Business, The University of Chicago [2002]. Used with permission. All rights reserved. www.crsp.uchicago.edu.

[6]We select funds that have either ICDI objective "Aggressive Growth", "Growth and Income" or "Long-Term Growth" or Strategic Insight objective "Aggressive Growth", "Growth & Income", "Growth", "Income Growth", "Growth MidCap", or " Small Company Growth". When both ICDI and Strategic Insight objective codes were missing, we selected funds with Wiesenberger objective "Growth and Current Income", "Long-Term Growth", "Maximum Capital Gains" or "Small Capitalization Growth".

Insight objective codes. Merging the CRSP dataset by ticker with Morningstar's April 1999 Principia Pro Database, we also obtained fund Morningstar objectives as of 1999[7]. In order to make our results comparable with those of earlier studies, we examine "local" tournaments within fund categories based on the Morningstar objective. The results based on ICDI and SI objectives are qualitatively similar and available under request. In addition, we consider the "global" tournament for the top rankings relative to the whole class of diversified US equity funds, since these rankings appear an important determinant of the fund flows (see Goriaev, 2003) and marking up effects (see Carhart, et al., 2002).

The number of funds in the sample grew from 271 in 1976 to 3731 in 1999, so that we have 22,345 fund-year observations. Table 1 reports summary statistics of the overall sample as well as Morningstar objective categories (aggressive growth, growth, growth-and-income, equity-income, and small company) in 1976-1999. During the sample period, diversified equity funds realized an average monthly return of 1.5% with the standard deviation of about 4.6% per year. In line with expectations, aggressive growth, growth and small company funds took higher total and unsystematic risks and achieved higher return than less risky growth-and-income and equity-income funds. In all categories, funds on average underperformed after adjusting for the market risk. Jensen's alpha ranges from -0.12% to -0.39% per year for growth-and-income and small company funds, respectively. On average, funds had a ten-year performance record and controlled about $508 million in assets; the largest and oldest funds were in the growth-and-income category.

To illustrate the difference between interim and end-of-the-year rankings, Table 2 reports the nine-month return rankings of funds that had top performance over the calendar year. As expected, funds highly ranked after the first three quarters of the year are most likely to top the annual rankings. For example, in the growth category, the top interim performer became the winner of the annual tournament in eleven years out of twenty four. However, sometimes funds ranked as low as 101 out of 239 or 52 out of 114 topped the annual rankings. Thus, the contest for the top annual ranking is not limited to a few funds with best year-to-date performance, and even funds ranked relatively low at the interim stage still have a chance to win the annual tournament.

6.2 Tested hypothesis

Assume that manager i has an objective function given in (4), i.e., he receives a bonus if fund i's two-period return is the highest among N funds in the category. In this case,

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[7]For funds that were not present in the Morningstar database, we determined Morningstar objectives on the basis of Strategic Insight objectives. The funds that disappeared prior to 1992 were assigned objectives on the basis of their investment policy, which had to be Common Stock, and their name.

given the information about the first-period fund performance (denoted Info$_1$, hereafter), the objective of manager i is to choose the amount of risk in the second period so as to maximize

$$E(C_i|\text{Info}_1) = B\text{Prob}\left(R_{i,1}R_{i,2} > \max_{j \neq i} R_{j,1}R_{j,2}|\text{Info}_1\right)$$

where $R_{i,t}$ is fund i's return in period t. The higher fund i's interim relative performance, the higher the probability of fund i outperforming the other funds at the end of the second period and receiving the bonus. In case of two funds, a fund's interim relative performance can be described by one variable: the difference between its own return and the return of the competing fund. Our theoretical model predicts that the fund's total risk in the second period decreases in this variable (see Proposition 1). In case of $N > 2$ funds, fund i's choice of risk in the second period depends on $N - 1$ variables: the differences between fund i's return and the returns of other funds over the first period. In Section 5, using simulations, we provided evidence that a general negative relationship between the fund relative performance in the first period and the total risk chosen in the second period holds in case of more than two funds.

For the empirical analysis, the $(N - 1)$-dimensional information about the relative performance of a fund over the first period (Info$_1$) will be summarized by one interim relative performance measure. For the sake of robustness, we use several different specifications of this measure. All of them are non-decreasing functions of the differences between fund i's return and the returns of other funds over the first period, which is taken to be the first $l \geq 6$ months of the year (in the main specification, we take $l = 9$ and examine fund strategic behavior during the last quarter of the year).

Our first measure is fund i's interim objective category return rank defined as

$$RANK_{i,1}^{OBJ} = \frac{1}{N-1} \sum_{j=1}^{N} I_{\{R_{i,1} > R_{j,1}\}},$$

where $I_{\{\}}$ is an indicator function and N is the number of funds with the same Morningstar objective as fund i. By construction, $RANK$ ranges from 0 for the worst interim performer to 1 for the top interim performer in the category.

The second measure we use is fund i's interim category-adjusted return:

$$RADJ_{i,1}^{OBJ} = R_{i,1} - R_1^{OBJ},$$

where R_1^{OBJ} represents the median return over the first l months of the year in the fund i's objective category.

Our third variable (denoted $PROB_{i,1}^{OBJ}$) measures the probability of fund i finishing the year ranked first in its category (i.e., having the highest annual return in the category), conditional on its interim performance, provided that funds do not change their strategies

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in the second part of the year and that market conditions do not change. Since we cannot calculate the probability of fund i having the maximum two-period return analytically, we estimate this probability from simulations. The simulation procedure is based on the market model and fund-specific parameters estimated during the first l months of the year (see Appendix C for a detailed description). By construction, $PROB^{OBJ}$ lies strictly between 0 to 1 and is increasing with fund's interim performance.

The last two variables measure fund's interim position in the "global" tournament for the top annual rankings. $RANK^{CLASS}$ is an interim return rank with respect to an asset class of all diversified equity funds, defined similarly to $RANK^{OBJ}$. $RADJ^{CLASS}$ is an interim asset class-adjusted return (fund i's return over the first l months of the year in excess of the median return over the same period among all diversified equity funds).

In our empirical analysis, we examine whether a fund's choice of risk in the last $12 - l$ months of the year is negatively related to its interim relative performance measured by the five variables defined above.

6.3 Methodology

The standard methodology for testing the tournament behavior of mutual funds compares the change in total risk between the first l months and the last $12 - l$ months of the year with interim l-month performance (see, e.g., Brown, Harlow, and Starks, 1996, and Koski and Pontiff, 1999). As demonstrated in Busse (2001) and Goriaev, Nijman, and Werker (2003), this methodology does not produce significant results after accounting for the cross-correlation effects in fund returns. Therefore, in this paper we develop a new, more powerful empirical methodology to examine strategic changes in fund risk.

Suppose that asset returns are generated from some factor model. A manager can influence the level of the fund's total risk in two ways: by changing the fund's factor loadings or the level of the idiosyncratic risk. Testing the model's predictions about the total risk, we should take into account that the fund's total risk may increase or decrease due to the change in factor volatility even when its factor betas remain the same. Busse (2001) reports that about 90% of the change in fund standard deviation between the first six months and the last six months of the year arises from changes in the volatility of the common risk (market, size, book-to-market, and momentum) factors and only about 10% from the deliberate actions of fund managers. This will not invalidate tests based on total risk, if all funds have the same factor betas. However, when funds' factor loadings differ from each other, tests based on total risk may produce biased results. There is extensive evidence in the literature (see, e.g., Brown and Goetzmann, 1997) that there are consistent differences between the risk exposures (in particular, market betas) of diversified US equity

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funds that compose our sample and that these differences are significant not only across, but also within categories. This suggests that changes in fund risk due to fund managers' strategic actions can be better measured by the changes in fund systematic risk exposures, i.e., factor betas.

In this paper, we focus our analysis on the within-year strategic changes in fund exposures to the market factor, which appears to be the most important determinant of fund total risk. Since systematic risk constitutes about 80% of the fund's total risk measured as dispersion of monthly returns (see Table 1), an increase in the fund's market beta typically results in an increase in the fund's total risk. Due to limitations of our data (fund monthly returns), we do not investigate strategic changes in fund exposures to other risk factors and idiosyncratic risk. It should be stressed that, similarly to tests of the total risk, tests of the idiosyncratic risk should account for the differences in fund loadings with respect to the factors omitted from the factor model (e.g., size and momentum) to produce unbiased results.

In contrast to the previous studies that first measured fund betas in the first and second parts of the year and then regressed the change in estimated betas on the explanatory variables (see, e.g., Koski and Pontiff, 1999), we use a more subtle approach. This allows us to circumvent the measurement problem, which is especially severe in case of monthly data. Assume that fund returns over period t ($t = 1$ and $t = 2$ correspond to the first l months and the last $12 - l$ months of the year, respectively) are generated from a one-factor model with the market factor:

$$\tilde{R}_{i,t} = \alpha_{i,t} + \beta_{i,t}\tilde{R}_t^m + \varepsilon_{i,t}, \tag{6}$$

where $E(\varepsilon_{i,t}) = 0$ and $E(\varepsilon_{i,t}\varepsilon_{i,s}) = 0$ ($t \neq s$). $\tilde{R}_{i,t}$ and $\tilde{R}_t^m$ represent the fund i's return and the market return (value-weighted CRSP index) in excess of the risk-free rate (one-month T-bill rate) accumulated over period t, while $\alpha_{i,t}$ and $\beta_{i,t}$ denote the Jensen's alpha and market beta of fund i in period t, respectively.

According to our model, each fund i follows a consistent risk policy with constant beta $\beta_{i,1}$ in the first period. In the second period, fund i modifies its beta depending on its interim relative performance $PERF_{i,1}$:

$$\beta_{i,2} = \beta_{i,1} + \gamma PERF_{i,1} + u_{i,2}, \tag{7}$$

where $PERF$ is measured as $RANK$, $RADJ$, or $PROB$ over the first l months of the year. Substituting (7) to (6) for $t = 2$ and assuming that fund Jensen's alphas (managerial skills) do not change during the year (i.e., $\frac{1}{l}\alpha_{i,1} = \frac{1}{12-l}\alpha_{i,2}$), we obtain

$$\tilde{R}_{i,2} - \frac{12 - l}{l}\alpha_{i,1} - \beta_{i,1}\tilde{R}_2^m = \gamma PERF_{i,1}\tilde{R}_2^m + \varepsilon_{i,2}^*, \tag{8}$$

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where the residuals $\varepsilon_{i,2}^* = u_{i,2}\tilde{R}_2^m + \varepsilon_{i,2}$ are assumed to have zero expectation and be uncorrelated over time. We also assume that $u_{i,2}$ and $\varepsilon_{i,2}$ are uncorrelated, i.e., fund managers do not possess a timing ability.

Our inferences are based on a Fama-MacBeth approach, which allows us to circumvent the problem of cross-correlation in fund returns. Each year, we estimate parameters of the cross-sectional regression (8) using a two-stage procedure. First, we estimate $\alpha_{i,1}$ and $\beta_{i,1}$ on the basis of fund monthly returns during the first l months of the year. This allows us to compute the market-model residuals over the last $12 - l$ months of the year (the left-hand side of (8)), which would be obtained under the null hypothesis that funds do not change their systematic risk during the year. In the second stage, we estimate γ from (8). This procedure yields the time series of twenty four values of γ characterizing the annual tournaments from 1976 to 1999. We aggregate the results over time by computing the mean value of the γ's. A t-statistic for testing the null hypothesis that $\gamma = 0$ is the ratio of mean γ and its standard error.

In order to ensure the robustness of our results, we also used a simulation approach to obtain empirical p-values of annual and mean γ adjusted for the cross-correlation in fund returns. The procedure of simulating returns under the null hypothesis of no strategic risk-taking was constructed as in Goriaev, Nijman, and Werker (2003). For each month, we simulated the vector of fund returns from a multivariate normal distribution with a mean vector and variance matrix that were estimated from the observed monthly fund returns in a given year. The results for the model (8) based on empirical p-values were qualitatively similar to those based on the Fama-MacBeth approach.[8]

6.4 Empirical results

The results based on model (8) for different break-ups of the year ($l = 6$ to $l = 11$) provide strong evidence of the negative relationship between fund performance over the first part of the year and subsequent choice of systematic risk. The coefficients on all five relative performance measures are negative and significant (in most cases, at 1% level, but never below 5% level). The results are significant not only statistically, but also economically. For example, a 30% move (e.g., a move from 30th to 60th return percentile) in objective category or asset class rankings, a 7-8% change in fund category- or class-adjusted return over the first three quarters of the year are associated with a subsequent change of about 0.1 in the last-quarter beta. The same change in beta is also caused by a 13% change in three-quarter

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[8]One can also use a bootstrap approach, as in Busse (2001), to obtain empirical p-values accounting for the cross-correlation effects. In his sample, the bootstrapped p-values were not materially different from simulated p-values.

$PROB^{OBJ}$. This negative risk-performance relationship seems to be very persistent. The coefficients on the five interim performance measures are negative in seventeen to twenty three out of 24 annual tournaments. The sensitivity of fund market betas to their prior relative performance is especially high in the middle of the year, both in terms of economic and statistical significance, and somewhat declines towards the end of the year. Probably, funds lagging behind the interim leaders become pessimistic about their prospects of winning the annual tournament and pay less attention to the ranking objectives towards the end of the year. In addition, this may be partly explained by the window-dressing effects documented in, e.g., Carhart et al. (2002).

From now on, we choose $l = 9$ as the main specification and examine fund strategic behavior during the last quarter of the year in more detail. Table 4 reports results based on our model (8) estimated separately in each fund category. In line with the expectations, aggressive growth and growth funds react stronger to the ranking objectives than funds from more conservative categories, such as growth-and-income and equity-income. This difference is especially pronounced for $RANK^{OBJ}$ and $PROB^{OBJ}$ measures, whose coefficients are two to three times higher for aggressive growth funds than for equity-income funds. The results for small company funds appear to be less significant; this is probably due to the small number of funds in this category.

One may argue that the negative relationship between interim performance and subsequent change in systematic risk is due to the cash flows that concentrate among the best-performing funds and drive beta down (see, e.g., Koski and Pontiff, 1999). In order to ensure that our results are not driven by this effect, we include net relative flow over the last quarter of the year as an additional determinant of fund beta in (7) and obtain the following regression model:

$$\tilde{R}_{i,2} - \frac{12-l}{l}\alpha_{i,1} - \beta_{i,1}\tilde{R}_2^m = (\gamma PERF_{i,1} + \theta CF_{i,2})\tilde{R}_2^m + \varepsilon_{i,2}^*, \tag{9}$$

where $CF_{i,2} = (TNA_{i,2}/TNA_{i,1}) - (1 + R_{i,2})$ and $TNA_{i,t}$ is the total net assets of fund i at the end of period t. Panel A of Table 5 demonstrates that cash flows do not have a significant impact at change in fund systematic risk. If any, their impact seems to be positive, contrary to the stated hypothesis. All performance measures under consideration remain significant. This conclusion stays the same for alternative definitions of flows (e.g., net absolute flows).

Finally, we investigate whether fund strategic risk-taking policies are related to other fund characteristics, such as size and age. One may expect that strategic behavior is more pronounced among small and young funds, for which it should be easier to change the riskiness of the portfolios. To test this hypothesis, we include in the basic model (7) the interaction terms of a fund's interim performance with logs of its size and age:

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$$\tilde{R}_{i,2} - \frac{12-l}{l}\alpha_{i,1} - \beta_{i,1}\tilde{R}_2^m = (\gamma_0 + \gamma_1 \ln TNA_{i,1} + \gamma_2 \ln Age_{i,1})PERF_{i,1}\tilde{R}_2^m + \varepsilon_{i,2}^*. \quad (10)$$

Panel B of Table 5 demonstrates that the negative relationship between interim performance and subsequent systematic risk change seems to be less pronounced among old funds. The age-performance interaction coefficient is positive for all five interim performance measures and is significant for $RANK^{OBJ}$, $PROB^{OBJ}$, and $RANK^{CLASS}$ at 5% level. A twofold increase in fund age is associated with a decrease in the sensitivity of the last-quarter beta to fund's interim ranking ($RANK^{OBJ}$ or $RANK^{CLASS}$) by 5% to 6% and to $PROB^{OBJ}$ by approximately 21%. The difference in systematic risk policy during the last quarter of the year between small and large funds does not appear significant.

7 Conclusion

The nature of the competition in the money management industry generates relative performance objectives for mutual fund managers. In this paper, we study how ranking objectives (as in a tournament) influence portfolio decision of a fund manager. In a two-period setting, we show how interim ranking influences the riskiness of the investment strategy chosen by managers in both periods. In the first period, managers choose the same risk level but do not maximize their expected return. In the second period, the interim loser increases risk, while the interim winner decreases risk relative to the first period. Furthermore, the level of risk undertaken by the interim loser is increasing with the difference in interim performances. Using simulations, we also demonstrate that the negative relationship between interim performance and risk chosen in the second period holds in the case with more than two competing funds.

Then, we provide empirical evidence on fund managers' risk-taking behavior, which is consistent with relative performance objectives. Funds with higher interim performance relative to their peers (funds with the same objective or all other diversified US equity funds) decrease systematic risk in the second part of the year to a larger extent than funds lagging behind after the first part of the year. This is very robust phenomenon: the observed pattern is similar for several interim performance measures, stays stable over time, and is more pronounced among funds from riskier categories and young funds. Thus, our findings support the tournament hypothesis that was first tested for a mutual fund industry in Brown, Harlow, and Starks (1996). However, in contrast to the previous studies, our results are robust to cross-correlation in fund returns. This is due to the improvement in the methodology, which limits the analysis of fund risk-taking behavior to the *systematic* risk, but circumvents the measurement problem and increases the power of the tests. As

discussed extensively in Busse (2001) and Goriaev, Nijman, and Werker (2003), the standard tests of the tournament hypothesis with total, systematic, and idiosyncratic risk measures based on monthly or daily return data do not produce time-persistent results robust to the cross-correlation effects.

Our results suggest that investors may be better off taking into account not only relative performance, but also the absolute level of performance when selecting between funds. Such allocation rule would "linearize" managers' incentives and mitigate the adverse incentives of fund managers.

Appendix A: Proof of Propositions 1 and 2

Proof of Proposition 1: An equilibrium in pure strategies in the period 2 subgame is a pair (v_l^*, v_w^*) such that

$$\frac{\partial H_w}{\partial v_w}(v_w^*, v_l^*, \delta) = 0, \qquad \frac{\partial H_l}{\partial v_l}(v_l^*, v_w^*, \delta) = 0 \tag{11}$$

and

$$\frac{\partial^2 H_w}{\partial v_w^2}(v_w^*, v_l^*, \delta) < 0, \qquad \frac{\partial^2 H_l}{\partial v_l^2}(v_l^*, v_w^*, \delta) < 0. \tag{12}$$

The system of first-order conditions is equivalent to

$$\delta + m(v_w^*) - m(v_l^*) = 2(v_l^* + v_w^*)m'(v_w^*) \tag{13}$$

and

$$\delta + m(v_w^*) - m(v_l^*) = -2(v_l^* + v_w^*)m'(v_l^*). \tag{14}$$

Conditions (13) and (14) imply that

$$m'(v_l^*) = -m'(v_w^*). \tag{15}$$

Hence, $v_l - \hat{v}$ and $v_w - \hat{v}$ are of opposite signs.

Assume that $m'(v_w^*) < 0$. From (13), this implies that $m(v_w^*) < m(v_l^*)$. Given that the interim winner's objective is to maximize $F(G(v_w, v_l, \Delta))$, we deduce that he can increase his probability of winning the contest by choosing $v = v_l^*$. Therefore, there exists a deviation that increases the probability of winning the contest. Hence, there cannot be an equilibrium with $m'(v_w^*) < 0$.

We now show that if $m'(v_w^*) > 0$ then the system of equations (13) and (14) has a unique solution. Let

$$\mathcal{H}(v_w^*, v_l^*, \delta) = \delta + m(v_w^*) - m(v_l^*) - 2(v_l^* + v_w^*)m'(v_w^*).$$

Equation (13) implies that in equilibrium $\mathcal{H}(v_w^*, v_l^*, \delta) \equiv 0$. Now, since $m'(v_w^*) = -m'(v_l^*)$,

$$\frac{d\mathcal{H}}{v_w}(v_w^*, v_l^*, \delta) = -m'(v_w^*)\left(1 - \frac{dv_l^*}{dv_w^*}\right) - 2m''(v_w^*)(v_l^* + v_w^*)$$

with $\frac{dv_l^*}{dv_w^*} = \frac{m''(v_w^*)}{m''(v_l^*)}$. Since $m''(\cdot) < 0$, $m'''(\cdot) \geq 0$ and $v_l^* > v_w^*$, it implies that $\frac{dv_l^*}{dv_w^*} > 1$. Therefore, $\mathcal{H}$ is monotonically increasing in v_w^* with $\lim_{v_w^* \to \hat{v}} \mathcal{H}(v_w^*, v_l^*, \delta) = \delta$ and $\lim_{v_w^* \to 0} \mathcal{H}(v_w^*, v_l^*, \delta) < 0$. Therefore, the equation $\mathcal{H}(v_w^*, v_l^*, \delta) = 0$ has a unique solution and there exists a unique equilibrium such that $v_w^* < \hat{v} < v_l^*$. The proof that v_w^* and v_l^* are increasing and decreasing in δ, respectively, follows directly from $m'(v_l^*) = -m'(v_w^*)$ and the strict concavity of $m(\cdot)$.

From conditions (13) and (14), we deduce that

$$d\delta + m'(v_w)dv_w - m'(v_l)dv_l = 2(dv_w + dv_l)m'(v_w) + 2(v_w + v_l)m''(v_w)dv_w, \tag{16}$$

$$d\delta + m'(v_w)dv_w - m'(v_l)dv_l = -2(dv_w + dv_l)m'(v_l) - 2(v_w + v_l)m''(v_l)dv_l. \tag{17}$$

This implies that

$$m''(v_w)dv_w = m''(v_l)dv_l. \tag{18}$$

In turn, this implies that dv_l and dv_w are of opposite signs. Furthermore, from (15), (16) and (18), we obtain that

$$d\delta = dv_w \left[2(v_l + v_w)m''(v_w) + m'(v_w)\left(1 - \frac{m''(v_w)}{m''(v_l)}\right)\right]. \tag{19}$$

Given the assumption that $m'''(\cdot) > 0$ and the result that $v_l^* > v_w^*$ in equilibrium, it follows that $m''(v_w^*)/m''(v_l^*) < 1$. Hence, v_w^* and v_l^* are decreasing and increasing in δ, respectively.. $\qquad \square$

Proof of Proposition 2: Let $\delta_{ij} = r_{i,1} - r_{j,1}$ $(i, j = 1, 2, i \neq j)$. From the proof of Proposition 1, we know that a manager who is leading after the first period has a probability strictly larger than $1/2$ of winning the contest. Now, if manager 1 chooses $v_1 = \hat{v}$ in the first period, then for any $v_2 \neq \hat{v}$ chosen by manager 2, $\text{Prob}(\delta_{2,1} > 0) < 1/2$, while if manager 2 chooses $v_2 \neq \hat{v}$ in the first period, then $\text{Prob}(\delta_{2,1} > 0) = 1/2$. Hence, $\hat{v}$ is a best reply to $\hat{v}$.. $\qquad \square$

Appendix B: Correlated returns

In this appendix, we analyze the case in which managers choose among portfolios with correlated returns. To do so, we modify the model of Section 3 in the following way. Assume that a safe asset (S) with return normalized to 1, and two risky portfolios are available. These two portfolios (hereafter, p_a and p_b) have returns (R_a and R_b) independently and normally distributed with variances $v_a = \hat{v}$ and $v_b > \hat{v}$ and means $m_a = m(v_a)$ and $m_b = m(v_b)$ (with $m_a > 1$ and $m_b > 1$), respectively; the function $m(\cdot)$ and $\hat{v}$ being as defined in Section 3. Therefore, $m_b < m_a$.

Denote l the interim loser and w the interim winner. In the second period, manager j ($j = l, w$) chooses an allocation $(\theta_{aj}, \theta_{bj})$, θ_{aj} and θ_{bj} being invested in portfolio p_a and p_b, respectively, and $(1 - \theta_{aj} - \theta_{bj})$ being invested in asset S. It follows that the return of manager j in the second period is

$$R_{j,2} = 1 + \theta_{aj}(R_a - 1) + \theta_{bj}(R_b - 1).$$

For tractability, we restrict the set of choices to $\theta_{aj} \geq 0$, $\theta_{bj} \geq 0$ and $\theta_{aj} + \theta_{bj} < 1$. This implies that shortselling the safe asset or the two risky portfolios is forbidden.

The main difference with Section 3 is that now returns are correlated, and their covariance is endogenous:

$$\operatorname{cov}(R_{l,2}, R_{w,2}) = \theta_{al}\theta_{aw}v_a + \theta_{bl}\theta_{bw}v_b.$$

Let $R_{1w}/R_{1,l} = \Delta$.[9] It is straightforward that the best reply of the interim winner is to choose the same allocation as the loser since in such a case, he wins the contest with probability 1. Conversely, the objective of the interim loser is to choose an allocation that generates a return correlated as little as possible with the return of the interim winner. It follows that such a game has only equilibria in mixed strategies. For some of these equilibria, we can derive results about the relative amount of risk undertaken by the two managers.

Proposition 3 *Assume that $A = 0$ and consider any equilibrium such that* (i) *managers only invest in the risky portfolios (i.e., $\theta_{aj} + \theta_{bj} = 1$, $j = l, w$) and* (ii) *managers randomize between the two same allocations $(\theta_{aj}, \theta_{bj}) = (\theta', 1 - \theta')$ or $(\theta_{aj}, \theta_{bj}) = (\theta'', 1 - \theta'')$ ($j = l, w$) with $\theta' > \theta''$. Denote q_j the equilibrium probability that manager j chooses $\theta_{aj} = \theta'$. Then, in such an equilibrium, the interim loser takes, on average, more risk than the interim winner: the interim loser chooses θ' with a lower probability than the interim winner, i.e., $q_l < q_w$.*

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[9]In the previous sections, it was assumed that managers are identical ex-ante. Here, there always exists an equilibrium such that $\Delta = 1$ with

probability 1 given the two risky portfolios available. Therefore, $\Delta > 1$ requires that managers did not choose the same portfolio in period 1. One possibility is that they were heterogeneously informed in period 1, while this is not the case in period 2.

Proof: Consider any equilibrium that satisfies conditions *(i)* and *(ii)*. Given the equilibrium strategy of the interim loser (i.e., the probability q_l with which he chooses θ'), the interim winner is indifferent between the two pure strategies. This implies that

$$q_l\text{Prob}\left(R_{l,2}/R_{w,2} > \Delta|\theta_{aw} = \theta', \theta_{al} = \theta'\right) + (1-q_l)\text{Prob}\left(R_{l,2}/R_{w,2} > \Delta|\theta_{aw} = \theta', \theta_{al} = \theta''\right) =$$
$$q_l\text{Prob}\left(R_{l,2}/R_{w,2} > \Delta|\theta_{aw} = \theta'', \theta_{al} = \theta'\right) + (1-q_l)\text{Prob}\left(R_{l,2}/R_{w,2} > \Delta|\theta_{aw} = \theta'', \theta_{al} = \theta''\right). \quad (20)$$

Given that

$$\text{Prob}\left(R_{l,2}/R_{w,2} > \Delta|\theta_{aw} = \theta', \theta_{al} = \theta'\right) = \text{Prob}\left(R_{l,2}/R_{w,2} > \Delta|\theta_{aw} = \theta'', \theta_{al} = \theta''\right) = 0,$$

it follows that

$$q_l = \frac{\text{Prob}\left(R_{l,2}/R_{w,2} > \Delta|\theta_{aw} = \theta', \theta_{al} = \theta''\right)}{\text{Prob}\left(R_{l,2}/R_{w,2} > \Delta|\theta_{aw} = \theta', \theta_{al} = \theta''\right) + \text{Prob}\left(R_{l,2}/R_{w,2} > \Delta|\theta_{aw} = \theta'', \theta_{al} = \theta'\right)}.$$

Proceeding similarly, we find that

$$q_w = \frac{\text{Prob}\left(R_{l,2}/R_{w,2} > \Delta|\theta_{aw} = \theta'', \theta_{al} = \theta'\right)}{\text{Prob}\left(R_{l,2}/R_{w,2} > \Delta|\theta_{aw} = \theta', \theta_{al} = \theta''\right) + \text{Prob}\left(R_{l,2}/R_{w,2} > \Delta|\theta_{aw} = \theta'', \theta_{al} = \theta'\right)}.$$

Let $r_{2,i} = \log(R_{2,i})$ $(i = l, w)$ and $\delta = \log(\Delta)$. Then, $q_w > q_l$ is equivalent to

$$\text{Prob}\left(r_{l,2} - r_{w,2} > \delta|\theta_{aw} = \theta'', \theta_{al} = \theta'\right) > \text{Prob}\left(r_{l,2} - R_{w,2} > \delta|\theta_{aw} = \theta', \theta_{al} = \theta''\right).$$

This is equivalent to

$$\Phi\left(\frac{\delta - (\theta' - \theta'')(m_a - m_b)}{\sqrt{(\theta' - \theta'')^2(v_a + v_b)}}\right) < \Phi\left(\frac{\delta - (\theta'' - \theta')(m_a - m_b)}{\sqrt{(\theta' - \theta'')^2(v_a + v_b)}}\right),$$

where Φ is cdf of the standard normal distribution. Given that $\theta' > \theta''$, this last inequality always holds. $\square$

If managers do not buy the risk-free bond, then the larger θ_{bj}, the larger the amount of risk taken by manager j. Proposition 3 states that in any equilibrium such that managers do not buy the risk-free bond and choose among the same two allocations, the interim loser takes more risk than the interim winner, on average.

This result is different from Taylor (2000) for one main reason. Taylor considers an economy with a risk-free asset and *one* risky asset. It follows that an interim winner increasing risk also increases the expected return of his portfolio. He does not face a trade-off between increasing the variance and decreasing the expected return. Conversely, we consider a situation such that managers have the possibility to choose portfolios with low expected return and high variance .

We can derive further results on the interim loser's risk taking incentives.

Proposition 4 *Assume that $A = 0$ and the interim winner chooses a portfolio such that $\theta_{aw} + \theta_{bw} = 1$ (i.e., does not buy the risk-free bond). If $\theta_{aw} \leq 1/2$, then the best reply of the interim loser is $\theta_{al} = 1$. If $\theta_{aw} > 1/2$, then the best reply of the interim loser is $\theta_{bl} = 1$.*

Proof: Proceeding as in the previous section, one shows that the objective of interim loser is to maximize

$$H(\theta_{al}, \theta_{bl}, \theta_{aw}, \theta_{bw}, \delta) = \frac{-\delta + (\theta_{al} - \theta_{aw})(m_a - 1) + (\theta_{bl} - \theta_{bw})(m_b - 1)}{\sqrt{(\theta_{aw} - \theta_{al})^2 v_a + (\theta_{bw} - \theta_{al})^2 v_b}}$$

with respect to θ_{al} and θ_{bl} under the constraint that $\theta_{al} + \theta_{bl} \leq 1$. First, we show that there cannot be an interior solution to this problem. To see this, assume that the interim loser chooses $\theta_{bl} \in (0, 1)$.

$$\frac{\partial H}{\partial \theta_{al}} = \frac{(m_a - 1)(\theta_{bw} - \theta_{bl})^2 v_b - v_a(\theta_{al} - \theta_{aw})\left[(\theta_{bl} - \theta_{bw})(m_b - 1) - \delta\right]}{\left((\theta_{aw} - \theta_{al})^2 v_a + (\theta_{bw} - \theta_{al})^2 v_b\right)^{3/2}}.$$

Therefore, if $\theta_{bl} < \theta_{bw}$, for any θ_{al}, $\partial H/\partial \theta_{al} > 0$. It implies that the interim loser chooses $\theta_{al} = 1 - \theta_{bl}$. Now if $\theta_{bl} > \theta_{bw}$, then it implies that $\theta_{al} < \theta_{aw}$ (since, by assumption, $\theta_{aw} + \theta_{bw} = 1$).

$$\frac{\partial H}{\partial \alpha_{bl}} = \frac{(m_b - 1)(\theta_{aw} - \theta_{al})^2 v_a - v_b(\theta_{bl} - \theta_{bw})\left[(\theta_{al} - \theta_{aw})(m_a - 1) - \delta\right]}{\left((\theta_{aw} - \theta_{al})^2 v_a + (\theta_{bw} - \theta_{al})^2 v_b\right)^{3/2}}.$$

If $\theta_{al} < \theta_{aw}$, then for any α_{al}, $\partial H/\partial \theta_{bl} > 0$. It implies that the interim loser chooses $\theta_{bl} = 1 - \theta_{al}$. Therefore, we always have $\theta_{aw} + \theta_{al} = 1$.

This implies that the problem of the interim loser is to choose θ_{al} to maximize

$$K(\theta_{al}, \theta_{aw}, \delta) = \frac{-\delta + (\theta_{al} - \theta_{aw})(m_a - m_b)}{\sqrt{(\theta_{aw} - \theta_{al})^2 (v_a + v_b)}}$$

under the constraint that $\theta_{al} \in [0, 1]$. It is straightforward that this is equivalent to maximizing $|\theta_{al} - \theta_{aw}|$. Therefore, if $\theta_{aw} < 1/2$, the interim loser chooses $\theta_{al} = 1$, while if $\theta_{aw} > 1/2$, the interim loser chooses $\theta_{bl} = 1$. $\square$

This proposition tells us that the best reply of the interim loser to an allocation of only risky portfolios by the interim winner is to choose the allocation of only risky portfolios that minimizes the correlation with the return of the interim winner.

From Propositions 3 and 4, we derive the following result.

Proposition 5 *Assume that $A = 0$. There exists an equilibrium such that*
(i) the interim winner chooses $\theta_{aw} = 1$ with probability q_w and $\theta_{bw} = 1$ with probability $(1 - q_w)$
(ii) the interim loser chooses $\theta_{al} = 1$ with probability q_l and $\theta_{bl} = 1$ with probability $(1 - q_l)$
(iii) $q_w > q_l$.

Proof: As already mentioned, the best reply of the interim winner is to play the same strategy as the interim loser. Furthermore, from Proposition 4, we know that $(\theta_{al}, \theta_{bl}) = (1,0)$ is a best reply to $(\theta_{aw}, \theta_{bw})$ with $\theta_{aw} > 1/2$ and $\theta_{aw} + \theta_{bw} = 1$; and that $(\theta_{al}, \theta_{bl}) = (0,1)$ is a best reply to $(\theta_{aw}, \theta_{bw})$ with $\theta_{aw} < 1/2$ and $\theta_{aw} + \theta_{bw} = 1$. This implies that there exists an equilibrium in which manager j chooses $(\theta_{aj}, \theta_{bj}) = (1,0)$ with probability q_j and $(\theta_{aj}, \theta_{bj}) = (0,1)$ with probability $1 - q_j$ $(j = w, l)$. Proposition 3 implies that $q_w > q_l$. □

This proposition states that there exist equilibria such that Proposition 3 holds: when both the variance and the covariance of the portfolios are strategic variables, then, on average, the interim loser takes more risk than the interim winner. Hence, the results derived in Section 4 still hold (qualitatively) when returns are correlated and their covariance level is a strategic variable.

Appendix C: Simulation procedure for the third relative performance measure

The third interim relative performance measure used in this paper, $PROB_{i,1}$, is the estimate of the probability that fund i has the highest annual return in its category, conditional on fund performance over the first l months of the year and given that funds do not change their strategies and that market conditions do not change in the last $12 - l$ months of the year. This appendix describes the simulation procedure used to compute this measure.

We use a market model (6) as a basis for our simulations. We simulate fund returns over the last $12 - l$ months of the year using the distribution parameters estimated on the basis of fund monthly returns during the first l months of the year. Specifically, we estimate fund Jensen's alphas and market betas, the mean and variance of the excess market return, and the variance matrix of the market-model residuals (in order to preserve the cross-correlation structure of fund returns). The vector of simulated fund returns over the last $12 - l$ months of the year is then calculated as a function of fund Jensen's alphas and betas as well as randomly generated values of the excess market return and the market-model residuals.

Formally, for each category consisting of N funds, we simulate the $N \times 1$ vector of fund returns over the last $12 - l$ months of the year using the following formula:

$$\tilde{R}_2 = \frac{12 - l}{l}(\alpha_1 + \beta_1 \tilde{R}_1^m + e_1), \tag{21}$$

where α_1 and β_1 denote the $N \times 1$ vectors of Jensen's alphas and market betas of funds estimated over the first l months of the year, respectively. The excess market return $\tilde{R}_1^m$ is generated from a normal distribution with mean and variance calculated on the basis of monthly excess market returns in the first l months of the year. The vector of residuals e_1 is generated from a normal distribution with zero mean and variance matrix estimated on the basis of monthly market-model residuals in the first l months of the year. Note that since the excess market returns, Jensen's alphas and market-model residuals are calculated on the l-month basis, they should be normalized by $\frac{12 - l}{l}$ to obtain return over the second part of the year in (21). The simulated probability of becoming a top fund in the category is based on 1000 replications of this procedure.

Table 1

Summary statistics of diversified US equity funds

The table presents average monthly performance and a number of other fund characteristics calculated for the whole sample of diversified US equity funds and separately for the aggressive growth, growth, growth-and-income, equity-income, and small company categories over the period January 1976 - December 1999. Jensen's alpha, beta, and unsystematic risk are calculated on the basis of the market model. Total risk and unsystematic risk are measured as the standard deviation of fund returns and the return residuals in the market model, respectively.

	All funds	Ag. Gr.	Growth	Gr. Inc.	Eq. Inc.	Sm. Co.
Total return, %	1.46	1.62	1.54	1.34	1.13	1.47
Total risk, %	4.64	5.99	4.71	3.82	3.16	5.70
Jensen's alpha, %	-0.17	-0.13	-0.14	-0.12	-0.14	-0.39
Beta	1.00	1.20	1.04	0.88	0.71	1.11
Unsystematic risk, %	2.07	3.00	1.98	1.31	1.40	3.33
Size, $mln	507.50	385.9	491.48	733.50	586.36	220.73
Age, years	10.19	10.16	10.22	13.12	9.22	5.90

Table 2
Interim performance rankings of top funds

The table presents nine-month return rankings of funds with the highest annual returns in the asset class of diversifed US equity funds and in the objective categories (aggressive growth, growth, growth-and-income, equity-income, or small company) in each year from 1976 to 1999. The number of funds in a given category and in a given year is in the parentheses.

Year	All funds		Ag. Gr.		Growth		Gr. Inc.		Eq. Inc.		Sm. Co.	
1976	7	(271)	1	(37)	3	(128)	1	(78)	2	(14)	1	(14)
1977	1	(265)	1	(37)	1	(124)	1	(76)	1	(14)	3	(14)
1978	1	(260)	1	(38)	1	(121)	1	(74)	1	(13)	1	(14)
1979	3	(259)	2	(34)	4	(124)	1	(72)	1	(14)	5	(15)
1980	2	(255)	2	(31)	2	(123)	1	(72)	2	(14)	1	(15)
1981	1	(256)	1	(30)	1	(121)	1	(73)	1	(16)	2	(16)
1982	1	(265)	1	(30)	2	(125)	2	(76)	1	(16)	1	(18)
1983	14	(280)	1	(31)	9	(132)	1	(80)	2	(16)	4	(21)
1984	2	(315)	2	(36)	1	(148)	1	(83)	1	(18)	2	(30)
1985	1	(345)	1	(42)	1	(162)	8	(91)	4	(20)	2	(30)
1986	2	(389)	1	(44)	2	(181)	1	(104)	3	(23)	3	(37)
1987	24	(431)	14	(51)	11	(198)	2	(114)	2	(24)	5	(44)
1988	1	(511)	1	(61)	1	(223)	1	(137)	1	(33)	1	(57)
1989	1	(542)	4	(64)	9	(233)	1	(142)	4	(40)	1	(63)
1990	1	(591)	1	(72)	2	(267)	2	(158)	4	(16)	1	(78)
1991	1	(640)	1	(76)	1	(291)	2	(171)	1	(19)	1	(83)
1992	1	(853)	3	(81)	1	(349)	101	(239)	1	(73)	1	(111)
1993	1	(1042)	6	(55)	1	(476)	1	(278)	2	(82)	1	(151)
1994	2	(1368)	2	(77)	2	(588)	2	(358)	52	(114)	8	(231)
1995	1	(1707)	1	(95)	1	(731)	12	(446)	22	(131)	2	(304)
1996	5	(2064)	5	(122)	9	(867)	1	(528)	1	(154)	1	(393)
1997	1	(2593)	1	(154)	1	(1130)	4	(634)	1	(176)	1	(499)
1998	1	(3112)	1	(184)	5	(1359)	1	(730)	1	(209)	1	(630)
1999	2	(3730)	2	(221)	3	(1668)	1	(868)	1	(231)	1	(742)

Table 3

Relationship between fund interim performance and

subsequent choice of systematic risk for different break-ups of the year

The table documents the relationship between fund performance over the first $l \geq 6$ months of the year and subsequent change in systematic risk for the sample of diversified US equity funds in January 1976 - December 1999. For each break-up of the year $(l : 12 - l)$ and each interim performance measure, the table reports the mean γ, the corresponding t-statistic, and the number of years with negative γ's, based on the model (8).

		$6:6$	$7:5$	$8:4$	$9:3$	$10:2$	$11:1$
$RANK^{OBJ}$	Coefficient	-1.376	-0.681	-0.486	-0.313	-0.175	-0.186
	t-statistic	7.77	6.36	5.31	3.77	2.45	2.35
	# negative	23	22	20	20	18	19
$RADJ^{OBJ}$	Coefficient	-0.072	-0.033	-0.021	-0.013	-0.007	-0.007
	t-statistic	7.77	6.53	5.5	3.95	2.85	3.31
	# negative	22	21	20	20	17	19
$PROB^{OBJ}$	Coefficient	-4.429	-2.246	-1.496	-0.778	-0.443	-0.366
	t-statistic	6.18	5.04	4.77	3.71	2.22	3.63
	# negative	22	21	21	18	19	21
$RANK^{CLASS}$	Coefficient	-1.578	-0.756	-0.558	-0.363	-0.183	-0.209
	t-statistic	6.17	5.22	4.47	3.30	2.12	2.17
	# negative	21	21	20	19	19	19
$RADJ^{CLASS}$	Coefficient	-0.075	-0.034	-0.022	-0.014	-0.007	-0.007
	t-statistic	7.06	5.97	4.85	3.54	2.58	3.23
	# negative	22	21	20	19	18	19

Table 4

Relationship between fund performance over the first three quarters of the year and choice of systematic risk in the last quarter of the year for different objective categories

The table documents the relationship between fund performance over the first three quarters of the year and subsequent change in systematic risk for the aggressive growth, growth, growth-and-income, equity-income, and small company objective categories of diversified US equity funds in January 1976 - December 1999. For each interim performance measure, the table reports the mean γ, the corresponding t-statistic, and the number of years with negative γ's, based on the model (8).

		Ag. Gr.	Growth	Gr. Inc.	Eq. Inc.	Sm. Co.
$RANK^{OBJ}$	Coefficient	-0.493	-0.334	-0.258	-0.228	-0.232
	t-statistic	3.53	3.74	4.08	3.30	1.81
	# negative	18	19	21	20	17
$RADJ^{OBJ}$	Coefficient	-0.014	-0.013	-0.013	-0.014	-0.008
	t-statistic	3.79	3.65	4.27	3.07	1.92
	# negative	19	18	20	20	17
$PROB^{OBJ}$	Coefficient	-1.532	-0.722	-0.669	-0.386	-0.925
	t-statistic	5.04	1.33	2.46	2.34	3.80
	# negative	22	17	16	18	19
$RANK^{CLASS}$	Coefficient	-0.447	-0.342	-0.318	-0.329	-0.232
	t-statistic	3.74	3.70	4.12	2.50	1.89
	# negative	17	19	21	19	17
$RADJ^{CLASS}$	Coefficient	-0.014	-0.013	-0.013	-0.014	-0.008
	t-statistic	3.79	3.65	4.27	3.07	1.92
	# negative	19	18	20	20	17

Table 5

Relationship between fund performance over the first three quarters of the year and choice of systematic risk in the last quarter of the year controlling for additional fund characteristics

The table documents the relationship between fund performance over the first three quarters of the year and subsequent change in systematic risk, controlling for additional fund characteristics: cash flows (see Panel A) and size-performance and age-performance interaction effects (see panel B). For each interim performance measure, the table reports the mean coefficients, the corresponding t-statistics, and the number of years with negative coefficients, based on models (9) and (10), respectively. The sample consists of diversified US equity funds in January 1976 - December 1999.

| | | Panel A | | Panel B | | |
		$PERF$	CF	$PERF$	$PERF*\ln TNA$	$PERF*\ln Age$
$RANK^{OBJ}$	Coefficient	-0.324	0.002	-0.403	0.002	0.035
	t-statistic	3.91	1.43	4.07	0.42	2.54
	# negative	20	8	20	12	6
$RADJ^{OBJ}$	Coefficient	-0.014	0.002	-0.015	0.000	0.001
	t-statistic	4.10	1.44	3.87	0.18	0.90
	# negative	21	7	19	11	10
$PROB^{OBJ}$	Coefficient	-0.750	0.001	-2.015	0.003	0.621
	t-statistic	3.86	0.94	2.78	0.03	2.07
	# negative	20	14	19	11	9
$RANK^{CLASS}$	Coefficient	-0.376	0.001	-0.442	0.002	0.028
	t-statistic	3.39	1.09	3.81	0.52	2.14
	# negative	20	8	19	12	9
$RADJ^{CLASS}$	Coefficient	-0.014	0.002	-0.015	0.000	0.001
	t-statistic	3.62	1.14	3.24	0.85	0.60
	# negative	19	8	19	13	8

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