Reservation Value Disclosure, Unraveling and Commitment.*

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Abstract

Most auctions, private or public, are conducted following a predetermined set of rules. While wider set of possible auction formats available to the seller may increase his expected revenue, choice of one of the formats reveals seller's private information; the seller may want to ex ante commit to an auction format in order to avoid the temptation to reveal private information ex post. The value of commitment is analyzed in the context of disclosed versus hidden reservation value choice. A policy of conditional disclosure is introduced, which generates revenue higher than that generated by either of the unconditional policies. In the context of public procurement auctions, implications of favoritism on the part of the auctioneer are analyzed.

1 Introduction.

Auctions that are used in practice, both in private and in public contexts, are almost universally conducted according to standard predetermined rules in one of the established

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and well known formats. Sellers are typically not free to choose any selling format they want but must follow one of the suggested patterns for soliciting bids and selecting winners.

Ex ante EBay lists only three selling formats for sellers to choose from, of which only one (called 'on-line auction') is a genuine auction in the economic sense of the word; the other two ('Fixed Price' and 'eBay Stores Inventory') amount to simply selling items at predetermined prices.[1] Once a seller selects 'on-line auction,' he does not have a freedom to choose between, for example, first price, second price or all pay auctions; neither is he allowed to discriminate between bidders in any way. Other auction sites have similar highly restrictive limitations on selling formats.

Rules governing public acquisition or procurement auctions are also highly restrictive. For example, Federal Acquisition Regulation, 'established to codify uniform policies for acquisition of supplies and services by executive agencies,' is about 2000 pages long. Similarly, Contracting Policy of the Treasury Board of Canada is over 1500 pages long and EU Procurement Legislation, combines 15 different acts over 1000 pages long in total. Although most of provisions of above mentioned legislature do not specifically concern auction formats, explicit limitations on auction practices are present.

In view of observed restrictions on auction formats for both public and private sellers a natural question arises: why do such restrictions exist? What makes it desirable to restrict auction formats a priori? Would not it be natural to leave the choice of the format and rules of an auction to the seller?[2]

The objective of this paper is to show that it may be optimal for the seller to *not* have freedom of auction format choice. The basic intuition is that the choice of the format itself may serve as a signal to potential bidders of important characteristics of the seller that he may want to conceal. One way for the seller to avoid sending such a signal is to commit ex ante to a specified auction format, provided such commitment is common knowledge.

To make this argument, I present a model in which the choice of format is limited to

[1] There is one other available format but it only applies to real estate sales.

[2] Throughout the paper I am focusing on auctions to sell, not auctions to buy. All the arguments can be easily translated for auctions to buy.

one decision: to disclose the true reservation value or not. Without any claim to generality itself, this single dimension of discretion is sufficient to illustrate a much more general logic.

In my model, there is one seller and two bidders who compete for a single item via sealed bid first price auction. The seller has a reservation value for the item; buyers have valuations that are independent from each other and from the seller's reservation value. The seller may ex ante (before he learns his reservation value) choose to commit to a disclosure policy that specifies whether the seller must disclose the reservation value; one way to do it is to hire an agent (an auctioneer or an auction house) to conduct the auction on the seller's behalf, according to explicit disclosure rules. I show that it may turn out to be optimal for the seller to commit to such a policy. This argument could justify existence of predetermined auction rules, such as eBay selling procedures or public procurement regulations: these rules, when they are common knowledge, may help to enhance seller's ex ante revenue.

In the context of independent private values the famous result of Myerson ([12]) and Riley and Samuelson ([14]) applies, which shows that the optimal (in terms of revenue) auction is an auction (for example, a first price auction) with reservation price set at the optimal level (and disclosed). However, their result depends crucially on the presumption that the seller can commit to any format. In contrast, I assume that the seller can not commit to any reservation price other than his true reservation value. This assumption is natural at least in the public procurement context: a public agency (be it a school board, county officials or a federal office) acting on behalf of the public, will have a hard time explaining to the public why they have rejected a bid that was above the reservation value of the project for the public.[3] Since the agency is aware of such a possibility, it has limited power in departing from the true reservation value in announcing minimum bids; in my

[3]For example, in case of a uniform $[0, 1]$ distribution of private values (to which I limit my attention) the optimal (in the sense of maximizing the expected revenue) reservation price v^* equal $\frac{1+r}{2}$, where r is the true reservation value. In particular, the optimal reservation price is always above $\frac{1}{2}$. A bidder may submit a bid that is above r but below $\frac{1+r}{2}$ and when it is rejected, initiate a media campaign and accuse the agency for incorrect allocating of taxpayer's money.

model I assume the extreme situation where the only disclosed reservation price that the agency may commit to is the true reservation value. Therefore, the only discretion that the agency may potentially have is whether to disclose the reservation value or not, and I study whether it is optimal to leave this discretion to the agency, and, more generally, what the ex ante optimal disclosure rule is.

Although the relevance of the above assumption is more doubtful in a private auction context, the assumption that the only credible reservation price for the seller is the true reservation value can still be defended. One further argument in support of it is that the resulting auction procedure is renegotiation proof: if all submitted bids are below seller's reservation value, he has no incentive to further negotiate the sale. On the contrary, if the seller posts a reservation price above the true reservation value and no bid meets it, he may be tempted to arrange a side deal with one of the bidders. If such renegotiation is expected by bidders, they will take the prospect of it into account when choosing their bids, so the optimality of the initial auction will depend on the feasibility of side trading.[4]

Focus on independent private value setting allows me to abstract from issues related to transmitting of payoff relevant information from the seller to the bidders. Famous linkage principle by Milgrom and Weber ([11]) states that the seller can increase his payoff by revealing such information, as long as such information is verifiable. There are a number of studies that address the choice of auction format by the seller from the signaling point of view. Jullien and Mariotti ([5]) and Cai, Riley and Ye ([1]) study signaling by reserve price while Kremer and Skrzypacz ([6]) study signaling by the choice of auction format itself. In another related paper Peyrache and Quesada ([13]) study strategic information revelation choice of an intermediary who is better informed than the seller about the quality of the good. In the public procurement literature, the restrictions imposed on auctioneer's behavior are analyzed within a principal-agent framework, where the 'public' is viewed as the principal who hires an agent to procure on its behalf. The focus in these studies is inevitably made on the divergent interests between the seller (or the buyer in the

[4]For optimal auctions when the seller can not commit see McAfee and Vincent ([9]) and most recent Skreta ([15]).

procurement context) and the auctioneer. For example, Laffont and Tirole ([7]) start by assuming that the government and the agency have conflicting objectives in that the agency favors one of the bidders, but unlike the government the agency possesses information about non-price dimensions of each bid (referred to as quality of the good provided), so agency services are indispensable, and derive optimal restrictions to be imposed by the government on the auction design that the agency may choose. Vagstad ([16] and [17]) further develops their analysis and discusses the choice between centralized (with the government directly carrying out the auction) and decentralized (with the government creating a special agency to carry out the auction) environment. The tradeoff is that the agency has better information about the quality of the product that each firm offers but also may favor local firm over foreign one, which causes inefficiencies. None of the papers that I am aware of highlight commitment benefits of having an auctioneer conduct the auction.

The message that I want to convey in this study is that the delegation of the auction format choice to the agency in charge of conducting the auction may not be optimal even if the interests of the government and the agency are perfectly aligned. The reason for that is that the government may want to commit to a specific procedure ex ante, so that the agency can not signal to the bidders project-specific information that it has before the bidding starts. Published instructions for carrying out procurement auctions serve as a commitment device for the auctioneer and this commitment has value.[5]

The rest of the paper is organized as follows. Section 2 describes the model and compares the revenue for the seller in two cases: under disclosed and hidden reservation value regimes. Section 3 discusses the value of commitment (predetermined auction format). Section 4 introduced conditional disclosure as an improvement over both disclosed value and hidden value regimes. In Section 5 I drop the assumption of the benevolent auctioneer and study what happens when one bidder is favored by the auctioneer. Section 6 concludes.

[5]Ferschtman et al ([3]) emphasize the commitment value of delegation contracts are publicly observed, albeit in a different context.

2 The Model.

There is a risk neutral seller who owns one indivisible good. He hires a risk neutral auctioneer to sell it for him, by means of a first price sealed bid auction. The good has reservation value r for the seller, but the seller does not know this reservation value until after he issues the instruction to the auctioneer. The seller's reservation value for the good is uniformly distributed on $[0, c]$, where $0 < c \leq 1$.[6]

The seller may issue instructions to the auctioneer whether or not to disclose r. If the auctioneer discloses r, it is verifiable.

For now I assume that the interests of the seller and the auctioneer are perfectly aligned: they both maximize the revenue from the auction (I relax this assumption in section 5).

There are $n \geq 2$ risk neutral bidders, their valuations are independent from each other and from seller's reservation value r and uniformly distributed on $[0, 1]$. They simultaneously submit sealed bids and the bidder who submits a high bid wins the good and pays his bid (at any equilibrium ties happen with zero probability, so it does not matter how they are broken). Each bidder maximizes her profit, $\pi = p \cdot v - b$, where v is her valuation, b is her bid and $p(b)$ is her probability of winning the good with bid b.

Here is the timing of the game:

1. The seller issues instructions to the auctioneer, prescribing him to disclose r or not, possibly contingent on r itself; the seller may choose to leave discretion to the auctioneer. These instructions become common knowledge.

2. The auctioneer learns r; he (credibly) discloses r if so instructed, keeps it hidden if so instructed and chooses whether to disclose it if the instruction leaves it to his discretion. If disclosed, r becomes common knowledge.

3. Bidders submit sealed bids.

[6] The assumption that $c \leq 1$ does not involve a loss of generality: it is easy to verify that bidders' equilibrium bidding functions are the same for $c > 1$ as for $c = 1$.

4. The bids are opened and the high bidder gets the good and pays her bid, provided her bid is greater than or equal to r.

I solve first for the equilibrium bidding functions and expected payoffs (to the seller and to each bidder) in two benchmark cases, when the auctioneer is instructed never to disclose r or always to disclose r. Superscript D stands for disclosed reservation value, and superscript ND stands for hidden reservation value.

Proposition 1 *The symmetric equilibrium bidding function for disclosed reservation value r is*

$$b^d(v,r) = \begin{cases} 0, & v \leq r \\ \frac{n-1}{n}v + \frac{r^n}{nv^{n-1}}, & v > r. \end{cases} \tag{1}$$

This equilibrium $b(v)$ is unique for $v \geq r$.

Proof See Appendix.

Given equilibrium bidding strategies $b^D(v,r)$ it is straightforward to calculate expected seller's expected revenue as a function of his reservation value r:

$$R^D(r) = \frac{n-1}{n+1} + r^n - \frac{n-1}{n+1}r^{n+1}. \tag{2}$$

Its average with respect to r is

$$R^D = \frac{n-1}{n+1} + \frac{c^n}{n+1} - \frac{n-1}{(n+1)(n+2)}c^{n+1}.$$

Now consider the alternative regime, in which the auctioneer does not disclose reservation value r.

Proposition 2 *The symmetric equilibrium bidding function, for nondisclosed reservation value, is*

$$b^h(v) = \begin{cases} \frac{n}{n+1}v, & v \leq \frac{(n+1)}{n}c, \\ \frac{n-1}{n}v + \frac{(n+1)^{n-1}}{n^{n+1}}\frac{c^n}{v^{n-1}}, & v > \frac{(n+1)}{n}c. \end{cases} \tag{3}$$

Proof is similar to that of Proposition 1 and is omitted.

Formula (3) is easy to interpret. When bidder's valuation v is small, she effectively competes with n rivals, namely, the other $n-1$ bidders and the seller. For uniform distribution this implies optimal bid equal to share $\frac{n}{n+1}$ of the valuation. In contrast, once a bidder is prepared to bid at least c, the seller is no longer a competitor for her; this results in the same differential equation (with slightly different initial condition) as in Proposition 1. Note also that for $c > \frac{n}{n+1}$ only the first (linear) fragment of the bidding function is present.

Expected revenue of the seller, as a function of his reservation value r, is straightforward to calculate; it is equal to

$$R^{ND}(r) = \begin{cases} \frac{n-1}{n+1} + \frac{(n+1)^{n-1}}{n^n} r^{n+1} + \frac{(n+1)^{n-1}}{n^n} c^n - \frac{(n+1)^{n-1}}{n^n} c^{n+1}, & c \leq \frac{n}{n+1}, \\ \frac{n^2}{(n+1)^2} + \frac{(n+1)^{n-1}}{n^n} r^{n+1}, & c > \frac{n}{n+1}. \end{cases} \tag{4}$$

The average expected revenue is

$$R^{ND} = \begin{cases} \frac{n-1}{n+1} + \frac{(n+1)^{n-1}}{n^n} c^n + \frac{(n+1)^n}{(n+2)n^n} c^{n+1}, & c \leq \frac{n}{n+1}, \\ \frac{n^2}{(n+1)^2} + \frac{(n+1)^{n-1}}{(n+2)n^n} c^{n+1}, & c > \frac{n}{n+1}. \end{cases} \tag{5}$$

Comparing (2) to (5) one establishes the following

Proposition 3 *Ex ante the seller always prefers never to disclose r to always disclosing r.*

Proof See Appendix.

The outcome of the auction is efficient if the reservation value is disclosed but not necessarily so if it is not, since in the latter case with positive probability the seller keeps the good even though there is a buyer with valuation higher than his reservation value. In either case, only a bidder with the highest valuation can get the good.

3 Value of Commitment.

With the help of the Propositions 1 and 2 I can now address the value of committing to a disclosure policy. The question that I address in this section is what the seller would prefer: to leave the choice of the auction format (disclosed vs. hidden reservation value) to the auctioneer's discretion, or to prescribe a specific disclosure policy to the auctioneer. Typically, ex ante restrictions on auctioneer's choice are justified by moral hazard arguments, that is assuming that the auctioneer's objective are different from those of the seller, i.e., the auctioneer may not in fact be maximizing seller's payoff. In this section I show that in fact ex ante restrictions on the information disclosure may be optimal even if the interests of the seller and the auctioneer are perfectly aligned. When the seller's reservation value realization is high, the auctioneer is tempted to disclose it in order to avoid being pooled with sellers with lower reservation values; standard unraveling argument in the auctioneer always disclosing the reservation value, which is suboptimal as shown in Proposition 3. Commitment not to disclose the reservation value even when it is high results in bidders bidding more aggressively which ultimately increases ex ante revenue on average. The following proposition adjusts the famous result of Grossman and Hart ([4]) to the auction setup.

Proposition 4 *When the decision whether to disclose the reservation value is left to the discretion of the auctioneer, the only subgame perfect symmetric Bayesian equilibrium involves disclosing reservation value r for any level of r.*

Proof. Assume the converse and consider an equilibrium in which the reservation value is not disclosed for some values of r. Denote by r_M the maximal of such values.[7] I now show that when the actual realization of the reservation value is r_M, the seller is strictly better off when it is disclosed.

[7]The proof presented here draws on the assumption that the set of reservation values which are not disclosed in equilibrium is closed and hence such maximum exists; the proof is easily generalized for arbitrary set.

If the reservation value is not disclosed, there are two possibilities: either in a symmetric equilibrium bidders will always be bidding below r_M and the object will not be sold (in which case the assertion is trivial, as disclosing r_M will definitely improve expected payoff to the seller) or in a symmetric equilibrium bidders will be bidding r_M at some valuation $\alpha r_M < 1$. Note that α can not be lower than 1 since then a bidder with valuation αr_M will be making negative profit on average. Neither can α be equal to 1, since then a bidder with valuation r_M will be making zero profit and bidding $r_M - \varepsilon$ for small enough $\varepsilon > 0$ will yield him positive profit on average: he will be winning at least ε if both seller's reservation value and all other bidders' bids are below $r_M - \varepsilon$, which has positive probability in equilibrium for small enough ε. Therefore, the only case to be considered is $\alpha > 1$.

The proof of Proposition 1 can be used to establish that the symmetric equilibrium bidding function for $v \geq \alpha r_M$ is $b(v) = \frac{n-1}{n}v + \frac{(\alpha r_M)^{n-1}}{v^{n-1}} r_M \left[1 - \frac{n-1}{n}\alpha^*\right]$. The seller's expected revenue in this case equals

$$\frac{n-1}{n+1} + n r_M (\alpha r_M)^{n-1} + \frac{(n-1)n}{n+1}(\alpha r_M)^{n+1} - (n-1)r^*(\alpha r^*)^n - (n-1)(\alpha r^*)^n. \tag{6}$$

On the other hand, if the seller discloses r_M, his expected revenue, as shown above, equals

$$R^D(r_M) = \frac{n-1}{n+1} + r_M^n - \frac{n-1}{n+1} r_M^{n+1} \tag{7}$$

It is easily verified that for $\alpha r_M > 1$ value (7) exceeds value (6). Therefore, it is not an equilibrium strategy for the seller not to disclose r_M, which contradicts the assumption that r_M is not disclosed. This contradiction completes the proof.

Therefore, the auctioneer, if he shares seller's objectives, will ex post choose to disclose the reservation value in any subgame perfect Bayesian equilibrium, unless ex ante committed not do so. However, as I showed in the previous section, always disclosing the reservation value is not optimal ex ante, for small enough spread of reservation values c; hence, the seller will find it optimal to ex ante limit auctioneer's discretion. This may be a reason why the restrictions on the choice of auction format exist in the first place.

4 Conditional Disclosure.

In this section I suggest a disclosure policy that is superior to both never disclosing the reservation value and always disclosing it. The main result of this section is

Theorem 1 *There exist a policy which dominates both discretional policy and the policy of no disclosure. It involves disclosing reservation value when it is above some y^* and not disclosing it when it is below y^*.*

Proof: For disclosed reservation values $r > y$, Proposition 1 applies. The symmetric equilibrium bidding function for disclosed reservation value $r \geq y$ is

$$b_y(v, r|r > y) = \begin{cases} 0, & v \leq r \\ \frac{n-1}{n}v + \frac{r^n}{nv^{n-1}}, & v > r. \end{cases} \tag{8}$$

The revenue of the seller, as a function of his reservation value r, equals

$$R_y(r|r > y) = \frac{n-1}{n+1} + r^n - \frac{n-1}{n+1}r^{n+1}.$$

If r is not disclosed prior to the bidding, the bidders realize that it is below y and update prior distribution $r \sim U[0, c]$ to posterior $r \sim U[0, y]$. Therefore, Proposition 2 applies and the expressions below follow.

Symmetric equilibrium bidding functions are

$$b(v|r \leq y) = \begin{cases} \frac{n}{n+1}v, & v \leq \frac{(n+1)}{n}y, \\ \frac{n-1}{n}v + \frac{(n+1)^{n-1}}{n^{n+1}}\frac{y^n}{v^{n-1}}, & v > \frac{(n+1)}{n}y. \end{cases} \tag{9}$$

Assuming that $y \leq \frac{n}{n+1}$, the expected revenue of the seller, as a function of his reservation value $r < y$, equals

$$R_y(r|r \leq y) = \frac{n-1}{n+1} + \frac{(n+1)^{n-1}}{n^n}r^{n+1} + \frac{(n+1)^{n-1}}{n^n}y^n - \frac{(n+1)^{n-1}}{n^n}y^{n+1}$$

In total, ex ante expected revenue of the seller equals

$$R_y = \int_0^y R_y(r|r \leq y)dr + \int_y^c R_y(r|r > y)dr = \frac{n-1}{n+1} + \frac{c^n}{n+1} - \frac{n-1}{(n+1)(n+2)}c^{n+1}$$
$$+ \left[\frac{(n+1)^{n-1}}{n^n} - \frac{1}{n+1}\right]\frac{y^{n+1}}{c} + \left[\frac{n-1}{(n+1)(n+2)} - \frac{(n+1)^n}{(n+2)n^n}\right]\frac{y^{n+2}}{c} \tag{10}$$

At $y = 0$ and $y = c$ the last expression coincides with those for disclosed and nondisclosed reservation values, respectively. It is maximized over y at

$$y^* = \frac{\frac{(n+1)^n}{n^n} - 1}{\frac{(n+1)^n}{n^n} - \frac{n-1}{n+1}}.$$

Proposition 5 *Optimal threshold reservation value y^* is lower than $\frac{n}{n+1}$.*

Proof See Appendix.

Therefore, for $c \leq y^*$ it is optimal for the seller to never disclose the reservation value; for $c > y^*$ the optimal strategy is to keep the reservation value secret if it is below y^* and to disclose it otherwise. ■

The intuition behind the result of this section is that when the actual reservation value is high, an uninformed bidder is likely to bid below it while her valuation is actually above it (because she averages her bid over the entire range of possible values of the reservation value) and disclosing the reservation value can help sell the good which otherwise may be unsold. On the other hand, when the reservation value is low, uninformed bidders are likely to bid above it anyway, and not disclosing it results in more aggressive bidding.

In the next two sections I depart from the assumption that the auctioneer has the same objective as the seller. I introduce favoritism on the part of the auctioneer and study how predictions of the above analysis change.

5 Favoritism.

In previous sections I have argued that it is optimal for the seller to ex ante commit not to disclose his reservation value; in the private context the mechanism for such a commitment is provided by preestablished restrictions on auction format of a particular auction house. In a public (e.g., procurement) auction context this commitment is achieved by hiring an auctioneer who conducts the auction on seller's behalf.

Commitment benefits of having the auctioneer rather than the seller conduct the auction are established above; however, there can naturally be agency costs associated with hiring an agent. In the context of my setup I model these costs by assuming that the auctioneer favors one of the bidders by disclosing seller's secret reservation value to her. For the rest of the paper I assume that there are only two bidders ($n = 2$) of which one is favored by the auctioneer. In addition I impose technical assumption $c \leq \frac{5}{8}$.

Even if the auctioneer secretly discloses the seller's reservation value to one of the bidders, the argument of Proposition 4 still applies: there is no perfect Bayesian equilibrium in which the seller does not ex post want to disclose the reservation value to the uninformed bidder when this value is high. Therefore, as long as keeping the reservation value hidden (at least from one of the bidders) is superior to publicly disclosing it, the auctioneer's service is still of value. On the other hand, now there are also costs of these services: the bigger the range of reservation values which are supposed to be kept hidden, the higher the advantage of the favored bidder.

I assume that the seller is aware of this practice by the auctioneer, but the discriminated bidder may or may not be aware of it. It turns out that in both cases the conditional disclosure rule still dominates both full disclosure and no disclosure, albeit with different threshold valuations. This is established by the following two propositions (superscripts uf and ef stand for unexpected and expected favoritism, respectively).

Proposition 6 *If the uninformed bidder is not aware of favoritism on the part of the auctioneer, optimal conditional disclosure policy involves disclosing the reservation value when it is above* $y^{uf} \approx 0.47$ *and not disclosing it otherwise; the seller's average payoff is*

$$\pi_s^{uf} = \frac{1}{3} + \frac{3c^2}{4} - .7394592495c^3, \tag{11}$$

the informed (favored) bidder's average payoff is

$$\pi_{fb}^{uf} = \frac{1}{6} - \frac{3c^2}{8} + .3706359226c^3, \tag{12}$$

the uninformed bidder's average payoff is

$$\pi_{db}^{uf} = \frac{1}{6} - \frac{3c^2}{8} + .3176340828c^3. \tag{13}$$

Proof. See Appendix.

Proposition 7 *If the uninformed bidder is aware of favoritism on the part of the auctioneer, optimal conditional disclosure policy involves disclosing the reservation value when it is above* $y^{ef} \approx 0.50$ *and not disclosing it otherwise; the seller's average payoff is*

$$\pi_s^{ef} = \frac{1}{3} + \frac{16c^2}{25} - 0.541867c^3, \tag{14}$$

the informed (favored) bidder's average payoff is

$$\pi_{fb}^{ef} = \frac{1}{6} - \frac{8}{25}c^2 + 0.281467c^3, \tag{15}$$

the uninformed bidder's average payoff is

$$\pi_{db}^{ef} = \frac{1}{6} - \frac{8}{25}c^2 + 0.280548c^3. \tag{16}$$

Proof. See Appendix.

As it could be expected, profits of the seller and the uninformed bidder are lower (compared to hidden reservation value no favoritism case) while the profit of the informed bidder is higher. Note that the threshold is below that for no favoritism case, which for $n = 2$ equals $\frac{15}{23}$. The intuition behind this finding is clear: if the seller suspects that the auctioneer is going to favor one of the bidders, he worries that this favored bidder, upon learning r, will not compete aggressively when r is high but rather will just bid r leaving the seller with no profit. Publicly announcing r restores competition and ultimately improves seller's expected profit.

It follows from (11)-(13) that the efficiency of the auction (defined as the sum of the expected payoffs to the seller and both bidders) is lower than that for hidden reservation value. There are two kinds of inefficiency associated with keeping the reservation value hidden. The first kind of inefficiency is that it may not be the bidder with the higher valuation who gets the item. The second kind of inefficiency is that the item may remain in the seller's hands even though one or both of the bidders have valuations above seller's reservation value but fail to bid above it. Without favoritism on the auctioneer's part, bidder's strategies are symmetric and monotone in valuations, so the item, if sold, always goes to the more efficient bidder; on the other hand, the inefficiency of the second kind is high. If the auctioneer favors one of the bidders, the inefficiency of the second kind is partly remedied, but the inefficiency of the first kind is introduced. The analysis above shows that on the balance the efficiency declines, so favoritism is not justified from the efficiency standpoint.[8]

In the context of favoritism, one may also consider the possibility that the interests of the seller are associated with those of the favored bidder. If the seller is the government, one of the bidders is a domestic firm and the other bidder is a foreign firm, than the government may be more interested in having a domestic rather than a foreign firm to win the object. The government may actually prefer to forego some of its own revenue in favor of that of the domestic firm, and hence tacitly sponsor favoritism on the part of the auctioneer. Whether or not it will actually want to do so depends on the weight with which the government values profit of the domestic firm versus its own profit. If this weight is low, the government will not want the auctioneer to engage in favoritism, since government's revenue is lower under favoritism. On the other hand, it is easy to see that the sum of the expected profits of the seller and the favored bidder is higher with favoritism than without it. Therefore, if the government's concern about the profit of the domestic firm is high enough, it will prefer hidden reservation value with favoritism regime over both disclosed and hidden reservation value regimes.

It follows from (11)-(16) that the efficiency of the auction is higher for expected favoritism than it is for unexpected favoritism. This is the case because uninformed bidder

[8]If efficiency, rather than optimality, is the seller's priority, his optimal policy is to always disclose the reservation value, as this leads to fully efficient outcome. However, as I showed earlier, optimality motivation calls to keep the reservation value secret. If it so happened that efficiency gains from favoritism exceeded efficiency costs, then it is conceivable that the seller, driven by some mixed optimality and efficiency concerns, would be *interested* in the auctioneer secretly favoring one of the bidders. However, the above efficiency result precludes such possibility: unexpected favoritism is always detrimental for the seller, whatever his objectives.

now bids more conservatively and therefore it is less often the case that the uninformed bidder wins whereas the informed bidder has in fact higher valuation. Note also that the sum of profit of the seller and the informed bidder is not only lower than that for the case of unexpected favoritism, but also than the profit for hidden reservation value, although still above that for disclosed reservation value. Two conclusions follow from this last observation. First, if the seller favors one of the bidders but can not privately disclose secret information to her without the other bidder being aware of such information leak, the seller should not disclose the information (and keep the two bidders in symmetric positions). Second, even in the worse possible scenario for the seller, i.e., if the auctioneer is corrupt and that he is corrupt is publicly known, still it is in the seller's interest to engage in the relationships with the auctioneer (rather than to unconditionally mandate disclosure of his reservation value or, equivalently, sell the object on his own). Indeed, it is easy to verify that Proposition 4 still applies, which means that without the commitment (or auctioneer's services) the seller will in any subgame perfect equilibrium be disclosing the reservation value. This last observation implies that benefits from commitment outweigh costs of agency: even when the auctioneer is known for malpractice, it is worth for the seller to draw on his services.

It also follows from proofs of propositions 6 and 7 that the discriminated bidder is bidding on average more conservatively if she is aware of the fact that the other bidder is being favored by the auctioneer.[9] A priori it is conceivable that the seller (especially when the seller is the government concerned not only with its own profits but also with that of the favored bidder) could find it in its interest to sponsor favoritism and to keep it common knowledge that one bidder is being favored. This, it turns out, is never the case: the seller does not benefit from favoritism and when favoritism takes place, it is not in the seller's interest to inform the discriminated bidder of favoritism.

[9] Bid of the uninformed bidder is the same as in the case of no strategic response for $v \leq \frac{6c}{5}$ and lower for $v > \frac{6c}{5}$.

6 Conclusion.

In this study I have shown that the services of an auctioneer who conducts an auction on behalf of the seller are valuable even if he has no advantages over the seller himself in terms of possessing relevant information or expertise. In the context of the decision whether to disclose the verifiable reservation value, I show that the seller will be better off is when he chooses the policy of never disclosing the reservation value. It is commonly argued that the main reason why the seller may not want to disclose it is the fear of collusion among bidders. In particular, when all the bidders collude, the seller will be making no profit if he discloses his reservation value, whereas if he keeps it secret he gets positive profit with positive probability. On the other hand, if the auctioneer is not supposed to reveal the information that he possesses, this opens the door for favoritism and corruption as long as one departs from the assumption of benevolent auctioneer, i.e., the auctioneer may still privately reveal the secret reservation price to one of the bidders, thus undermining the idea to keep it secret and, worse, creating asymmetry between bidders, potentially deteriorating seller's profit.

However, as the results of this study suggest, in the independent private value setting,[10] it is not necessarily true that the profit-maximizing seller is always better off disclosing the secret price rather than concealing it, under the important assumption that the seller's reservation value, once disclosed, is verifiable. In fact it turns out that in many cases the reverse is true: concealing the true reservation value may, while compromising efficiency of the allocation, improve the seller's revenue. Therefore, it is unnecessary to appeal to the threat of collusion between bidders to justify keeping the reservation price hidden.

Neither is it necessary, it turns out, to introduce moral hazard in seller-auctioneer relationships in order to justify instructions limiting auctioneer's discretion on whether to disclose the reservation price or not. In fact, the auctioneer and the seller, both maximizing seller's expected ex ante payoff, may find it in their interest to commit to specific rules

[10] If bidders' valuations are affiliated, that makes the case stronger for revealing the information rather than concealing it, as pointed out by Milgrom and Weber ([11]).

regarding information disclosure. This commitment (observed by bidders) keeps bidders from making adverse inferences from the fact that that the auctioneer keeps his reservation value hidden and induce them to bid aggressively enough that the seller's profit becomes on average higher than in the case of full auctioneer's discretion.

I further introduce the policy of conditional disclosure, i.e., disclosing the reservation value when it is above a certain threshold and not disclosing it otherwise; this policy is shown to be superior to both always disclosing and never disclosing the reservation value; I then showed that a conditional disclosure policy (although with lower threshold) is optimal if the seller expects favoritism on the part of the auctioneer. This last result resembles findings of Lizzeri ([8]), who shows, in context of unobservable quality of a good, that an informed intermediary will adopt the strategy of certifying that a good has at least some prespecified minimum quality. However, underlying assumptions that drive his result are very different.

While the basic model built in this paper suffices to illustrate the insights specified above, there are issues that are left behind. Effects of possible collusion among bidders are not studied, and neither is the mechanism of corruption modeled explicitly. Finally, an open question is how the findings of this analysis generalize to multiple dimensions of the seller's objectives (e.g., quality). These issues deserve further scrutiny.

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7 Appendix

Proof of Proposition 1. It is straightforward to show that the equilibrium symmetric bidding function $b(v)$ is monotonic in v for $v \geq r$.[11] Now, if bidder 1 believes that all other bidders are bidding according to strategy $b(\cdot)$ and the valuation of bidder 1 herself is v, she chooses her bid b to maximize $[b^{-1}(b)]^{n-1} \cdot (v-b)$, so at the optimum $b^{-1}(b) = (n-1)(v-b)(b^{-1}(b))'$. Since I am looking for symmetric equilibrium bidding function, at the equilibrium it must be the case that $b = b(v)$ and then $b^{-1}(b) = v$. By the inverse function theorem $(b^{-1}(b))' = \frac{1}{b'(v)}$, so that the differential equation for $b(v)$ is

$$b'(v) = \frac{v-b}{v}(n-1).$$

General solution to this differential equation is $b(v) = \frac{n-1}{n}v + \frac{k}{v^{n-1}}$. It is straightforward to see that at the optimum $b(r) = r$, so that $k = \frac{r^n}{n}$ and the expression for $b(v)$ follows.

Proof of Proposition 3. I have to show that R^{ND} is higher than R^D for any n and any $c \leq 1$.

Consider first the case when $c \leq \frac{n}{n+1}$. Then I have to show that

$$\frac{n-1}{n+1} + \frac{c^n}{n+1} - \frac{n-1}{(n+1)(n+2)}c^{n+1} < \frac{n-1}{n+1} + \frac{(n+1)^{n-1}}{n^n}c^n + \frac{(n+1)^n}{(n+2)n^n}c^{n+1},$$

[11] Bidding strategy for $v < r$ is irrelevant since there is no chance to earn positive payoff anyway. Without affect to anyone’s payoff I assume that $b(v,r) = 0$ for $r < v$.

which is equivalent to

$$c \leq \frac{(n+2)\left[(n+1)^n - n^n\right]}{(n+1)^{n+1} - (n-1)n^n}.$$

It is sufficient to show that

$$\frac{(n+2)\left[(n+1)^n - n^n\right]}{(n+1)^{n+1} - (n-1)n^n} \geq \frac{n}{n+1},$$

which is straightforward.

Next, consider the case when $c > \frac{n}{n+1}$. I have to show that

$$\frac{n-1}{n+1} + \frac{c^n}{n+1} - \frac{n-1}{(n+1)(n+2)}c^{n+1} - \left[\frac{n^2}{(n+1)^2} + \frac{(n+1)^{n-1}}{(n+2)n^n}c^{n+1}\right] \tag{17}$$

is negative. The derivative of expression (17) equals

$$c^{n-1}\left[\frac{n}{n+1} - \left[\frac{n-1}{n+2} + \frac{(n+1)^n}{(n+2)n^n}\right]c\right],$$

so it is clear that if there exists c such that (17) is positive, it should also be positive at

$$c^* = \frac{\frac{n}{n+1}}{\frac{n-1}{n+2} + \frac{(n+1)^n}{(n+2)n^n}}.$$

Plugging c^* into expression (17) and multiplying by $(n+1)^2$ results in

$$c^{*n}\left[n+1 - \left[\frac{(n+1)^{n+1}}{(n+2)n^n} + \frac{n^2-1}{n+2}\right]c^*\right] - 1 = c^{*n} - 1,$$

which is negative for $c^* < 1$, QED.

Proof of Proposition 5. Inequality $y^* < \frac{n}{n+1}$ can be easily reduced to $\left(\frac{n+1}{n}\right)^n < \frac{3n+1}{n+1}$. The lefthandside does not exceed e; the righthandside exceeds e for $n \geq 7$. For $n = 2, \ldots, 6$ the required inequality is verified by direct calculation.

Proof of Proposition 6. I calculate the best response to $b^h(v)$ on the part of the informed bidder as a function of her own valuation v and reservation value r that she learns, as well as the expected profit of the seller and each bidder.

If informed bidder's own valuation v is below seller's reservation value r, the informed bidder can not win the auction without making negative profits. Her exact bid is not important, provided it is below r; as above, I specify it at zero.

Consider the case when informed bidder's valuation v is above r. She can choose either $b \leq c$ or $b > c$. If she bids $b \leq c$ she will be winning the auction whenever her bid is above that of the uninformed bidder, equal to two thirds of the uninformed bidder's valuation (according to $b^h(v)$). Hence the probability that the informed bidder will win the auction if she bids $b \in [r, c]$ is equal to $\frac{3b}{2}$ and she chooses $b \in [r, c]$ to maximize her expected payoff $\frac{3b}{2}(v - b)$. If she decides to bid below c, she will choose $b(v) = r$ for $v \in [r, 2r]$ and $b(v) = \frac{v}{2}$ for $v > 2r$. Her expected payoff will be $\frac{3r(v-r)}{2}$ and $\frac{3v^2}{8}$.

If the informed bidder decides to bid above c, then her being informed about the reservation value is irrelevant and she does not have any advantage over the uninformed bidder. Hence, by virtue of $b^h(v)$ being the symmetric equilibrium in the case of no favoritism, informed bidder's optimal bidding strategy against $b^h(v)$ is $b^h(v)$ itself. She will be winning with probability v and receiving payoff of $v - b^h(v) = \frac{v}{2} - \frac{3c^2}{8v}$, so that her total expected payoff is $\frac{v^2}{2} - \frac{3c^2}{8}$.

To complete the description of the optimal strategy of the informed bidder, I must specify the cutoff point below which she bids $b \leq c$ and above which she bids $b > c$. This is done by comparing the expected payoffs derived above. It is easily verified that for small r (such that at $v = 2r$ bidding above c is not profitable) this cutoff equals $c\sqrt{3}$, while for large r it is $\frac{3r+\sqrt{3c^2-3r^2}}{2}$. Therefore, best response bidding function of the informed bidder takes the following form:

$$b^{uf}_{fb}(v, r) = \begin{cases} 0, & v \leq r, \\ r, & r \leq v < 2r, \\ \frac{v}{2}, & 2r \leq v \leq c\sqrt{3}, \\ \frac{v}{2} + \frac{3c^2}{8v}, & v > c\sqrt{3}. \end{cases} \tag{18}$$

for $r \leq \frac{c\sqrt{3}}{2}$, and

$$b^{uf}_{fb}(v,r) = \begin{cases} 0, & v \le r, \\ r, & r < v \le \frac{3r+\sqrt{3c^2-3r^2}}{2}, \\ \frac{v}{2} + \frac{3c^2}{8v}, & v > \frac{3r+\sqrt{3c^2-3r^2}}{2}. \end{cases} \tag{19}$$

for $r > \frac{c\sqrt{3}}{2}$. Expressions (11)-(13) directly follow.

Proof of Proposition 7. I now solve for the equilibrium pair of strategies. I start with the uninformed bidder. I conjecture that she follows a linear strategy for low valuations: $b^{ef}_{db}(v) = \lambda v$ for $v \le v^*$ (the value of v^* to be determined). If that is her strategy, then, for small enough r, the best response of the informed bidder is to bid r for $v \in [r, 2r]$ and $\frac{v}{2}$ for $v \in [2r, c]$. Uninformed bidder's optimization problem is then easily solved: her optimal bidding strategy is $b(v) = \frac{2v}{3}$ for $v \le v^*$, confirming the linearity conjecture.

Also, when v is large (higher than some v^{**} to be determined), both bidders will bid above c, in which case they are symmetric as the information that the favored bidder possesses has no value. In this case, as was shown above, the equilibrium bidding functions are $b(v) = \frac{v}{2} + \frac{2cv^{**}-(v^{**})^2}{2} \cdot \frac{1}{v}$.

I drop technical details that help to derive optimal bidding strategies in the medium range of v and also to solve for v^* and v^{**}. In equilibrium the bidding strategy of the uninformed bidder must be a solution to the differential equation shown below. I used numerical methods to solve for values v^* and v^{**}. It turns out that $v^* = \frac{6c}{5}$ and $v^{**} = \frac{8c}{5}$. Below are the equilibrium strategies for both the informed and the uninformed bidder.

The equilibrium strategy of the uninformed bidder is

$$b^{ef}_{db}(v) = \begin{cases} \frac{2v}{3} & v \le \frac{6c}{5}, \\ \tilde{b}(v) & \frac{6c}{5} < v \le \frac{8c}{5}, \\ \frac{v}{2} + \frac{8c^2}{25v} & v > \frac{8c}{5}, \end{cases} \tag{20}$$

where $\tilde{b}(v)$ is a monotonic function such that after substitution $x = \frac{5b}{4c}$ and $t = \frac{5v}{4c}$, function $x(t)$ is the solution to differential equation

$$x'(t) = \frac{(2t-x)\sqrt{1-2tx+t^2} + 2t^2 - 4tx + x^2 + 1}{2t\sqrt{1-2tx+t^2} + 3t^2 + 2 - 5tx}$$

with initial condition $x(\frac{3}{2}) = 1$. Function $x(t)$ is very well approximated by linear $\tilde{x}(t) = \frac{t}{2} + \frac{1}{4}$.

The equilibrium strategy of the informed (favored) bidder is

$$b_{fb}^{ef}(v, r|r \leq \frac{4c}{5}) = \begin{cases} 0 & v \leq r, \\ r & r \leq v < 2r, \\ \frac{v}{2} & 2r \leq v \leq \frac{8c}{5}, \\ \frac{v}{2} + \frac{8c^2}{25v} & v > \frac{8c}{5}, \end{cases} \tag{21}$$

for $r \leq \frac{4c}{5}$ and

$$b_{fb}^{ef}(v, r|r > \frac{4c}{5}) = \begin{cases} 0 & v \leq r, \\ r & r < v \leq \tilde{b}^{-1}(r) + \sqrt{\frac{16}{25}c^2 - 2r\tilde{b}^{-1}(r) + [\tilde{b}^{-1}(r)]^2}, \\ \frac{v}{2} + \frac{8c^2}{25v} & v > \tilde{b}^{-1}(r) + \sqrt{\frac{16}{25}c^2 - 2r\tilde{b}^{-1}(r) + [\tilde{b}^{-1}(r)]^2} \end{cases} \tag{22}$$

for $r > \frac{4c}{5}$.

Expressions (14)-(16) directly follow.